Vibration Analysis of Nonlinear-Dynamic Rotor-Bearing Systems and Defect Detection

Ph.D. Dissertation

Athanasios C. Chasalevris

Dipl. Mechanical Engineer

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This work is dedicated

to my parents, Christos and Agapi,
to my teacher Prof. Papadopoulos Christos

“The most beautiful thing we can experience is the mysterious. It is the source of all true art and all science. He to whom this emotion is a stranger, who can no longer pause to wonder and stand rapt in awe, is as good as dead: his eyes are closed” - Albert Einstein.
Examining Committee

1) Associate Professor Dr. –Eng. Papadopoulos Christos (supervisor),
Department of mechanical engineering & aeronautics, university of Patras

2) Professor Dr. –Eng. Fassois Spilios
Department of mechanical engineering & aeronautics, university of Patras

3) Associate Professor Dr. –Eng. Dentsoras Argiris
Department of mechanical engineering & aeronautics, university of Patras

4) Professor Dr. –Eng. Anifantis Nikolaos
Department of mechanical engineering & aeronautics, university of Patras

5) Professor Dr. –Eng. Spentzas Konstantinos
Faculty of mechanical engineering, national technical university of Athens

6) Associate Professor Dr. –Eng. Michailidis Athanasios
Department of mechanical engineering, Aristotle university of Thessalonica

7) Associate Professor Dr. –Eng. Chondros Thomas
Department of mechanical engineering & aeronautics, university of Patras
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Abstract

This work focuses in two main directions of rotor dynamics field, the simulation of rotor bearing systems and the fault diagnosis. From the serious multiple faults that can appear in a rotor bearing system two of them are the target of current research: the transverse fatigue crack of a rotor and the radial extended wear in a bearing. The transverse crack is a defect able to bring a catastrophic failure of the system when the growth (depth) takes high percentage values relatively to radius of the shaft (i.e. >60%) and the symptoms of crack presence have been widely investigated during last four decades yielding efficient methods for the early crack detection. On the other hand the defect of bearing wear is much less investigated without results connected with wear diagnosis methods. Concerning previous works in those two defects the current dissertation’s persuasion is firstly to make a proposal in bearing wear detection, secondly to achieve a method definition able to detect a breathing transverse crack in a different way from those referred to literature.

For the subject of crack detection, a different crack breathing model is proposed with emphasis in coupled local compliances definition and their variation during rotation while for the subject of bearing wear detection, a wear model from the literature is used with emphasis in rotor bearing system construction in a different way in relation to what up to now is available in literature. The rotor bearing system construction (simulation) is a matter widely investigated since early 60’s and some points of the current work try to differ in the way that the rotor and the fluid film bearings interact in discrete time. The concept of nonlinear fluid film forces is confronted in this work leaving out the nonlinear stiffness and damping bearing fluid film coefficients and assuming that during the journal whirling no equilibrium point must be defined in order to evaluate the future progress of vibration. Towards generality the fluid film bearings are not defined geometrically as short or long. These two specific geometric assumptions of short/long bearing appear widely in real machines and yield analytical expressions of fluid film forces but in current work the finite fluid film bearing is used demanding the well known finite difference method in order to evaluate the impedance forces, as many researches have propose.

Both defects are met in a rotor bearing system parted from a continuous rotor and finite fluid film bearings. An entire chapter is dedicated in the way that Rayleigh equation of rotor motion incorporates internal damping using exclusively Real number confrontation, and in the way that fluid film forces react in rotor motion by defining boundary conditions in every discrete time moment. The definition of boundary conditions in discrete time makes them functions of the entire system response yielding a nonlinear dynamic system with the resulting time
histories to be characterized from periodicity or quasi-periodicity sometimes depending in the defects presence.

An extended analysis of time histories of the intact and the defected system is made in order to invest the symptoms of each defect in magnitudes of time and frequency domain. Time-frequency analysis is performed using continuous wavelet transform in virtually or really (the former from simulation, the latter from experiment) acquired time histories in order to extract the variable coupling phenomenon exclusively due to the breathing crack from the other two main reasons of coupling, the bearings and the shaft. Vertical response due to crack coupling is amplified when the crack coupled compliances become larger under an electromagnetic horizontal excitation in the rotor. This rapid in time variable coupling due to crack is used at last in order to detect the crack presence. The external excitation is used also in the case of wear detection since results of time-frequency analysis yield unexpected amplification of specific harmonics when the wear defect is present.

Both considerations about the corresponding fault detection are tried in a real experimental system after the observation that response of the current rotor bearing simulation converges with the response of the physical system in characteristics that are judged important for the method robustness. The general speculation is that both defects have to be detected without the need of operation interruption since this cannot be feasible (high cost) in real turbo machinery plants and in an early growth that coincides with safe machine operation. The defect growths have to be at least 10% (of radius) for the crack and 20% (of radial clearance) for bearing wear so as the methods to be efficient.

*Keywords: fault detection, rotor bearing systems, nonlinear systems, breathing crack, cracked rotor, worn bearings, continuous rotor, coupled vibrations*
Το τα λιγότερο προσομοίωση στην έγκαιρη έδρανα μεθόδους αντικείμενο αντικείμενο δυναμική Η Abstract (in Greek) – Περίληψη

Η συγκεκριμένη διατριβή επικεντρώνεται κυρίως σε δύο κατευθύνσεις του αντικειμένου της δυναμικής των περιστρεφόμενων αξόνων: την προσομοίωση συστημάτων αξόνων και εδράνων και την ανίχνευση βλάβων σ’ αυτά τόσο σε αναλυτικό όσο και σε πειραματικό επίπεδο.

Από τις συνήθως απαντώμενες βλάβες σε τέτοια συστήματα δύο από αυτές αποτελούν στόχους για τη συγκεκριμένη εργασία: η εγκάρσια ρωγμή λόγω κόπωσης του άξονα και η ακτινική φθορά των εδράνων αλλάζουσα.

Η εγκάρσια ρωγμή είναι μία βλάβη ικανή να επιφέρει ολοκληρωτική καταστροφή της μηχανής στην οποία παρουσιάζεται, όταν η έκταση της υπερβαίνει το 60% περίπτου της διαμέτρου και τα συμπτώματα της ρωγμής στην ταλαντωτική συμπεριφορά του συστήματος έχουν εκτενώς διερευνηθεί τις τελευταίες δεκαετίες, με αποτέλεσμα την ανάπτυξη ποικίλων μεθόδων για την έγκαιρη ανίχνευση της βλάβης. Αντιθέτως, η φθορά των εδράνων αποτελεί μια βλάβη πολύ λιγότερο διερευνημένη συγκριτικά με τη ρωγμή, χωρίς αποτελέσματα για τη διάγνωση της κατά τη λειτουργία της μηχανής. Έχοντας υπ’ όψη της εργασίες των προηγουμένων ετών στο αντικείμενο της ανίχνευσης αυτών των δύο βλαβών, η παρούσα διατριβή έχει ως στόχο πρωτίστως να προτείνει μεθόδους για την ανίχνευση της φθοράς του εδράνου και δευτερεύοντας να επιτύχει την ανίχνευση της ρωγμής με ένα διαφορετικό ως προς τη φιλοσοφία, και απλό ως προς την εφαρμογή τρόπο, αναφορικά με τις μέχρι σήμερα μεθόδους.

Για το αντικείμενο της ανίχνευσης της ρωγμής, προτείνεται αρχικά μία διαφορετική προσομοίωση της συμπεριφοράς της κατά την περιστροφή με έμφαση στον υπολογισμό των τοπικών ενδοσκεύαστων σύζευξης κατά τη διάρκεια της λειτουργίας του συστήματος, ενώ για το αντικείμενο της ανίχνευσης της φθοράς χρησιμοποιείται ένα ήδη υπάρχον μοντέλο από τη βιβλιογραφία. Και οι δύο βλάβες ενσωματώνονται σε μία νέα ως προς τη βιβλιογραφία προσομοίωση συστήματος αξόνων και εδράνων η οποία αντιμετωπίζει τον άξονα και τα εδράνα ως ένα ενιαίο σύστημα χρησιμοποιώντας τις παι ακριβείς έως τώρα προσεγγίσεις ταλάντωσης συνεχώς μέσου και της υδροδυναμικής θεωρίας των εδράνων. Η προσομοίωση συστημάτων αξόνων και εδράνων είναι ένα ζήτημα ευρέως διερευνημένο από τις αρχές της δεκαετίας του ’60 και ορισμένα στοιχεία της παρούσας διατριβής, πάνω στην αντιμετώπιση του θέματος αυτού, έχουν ως στόχο τη βελτίωση με την παρούσα ανάλυση της υπάρχουσας γνώσης, για τον τρόπο που έδρανο και άξονας αλληλεπιδρούν σε διακριτό χρόνο.
Το θέμα των μη γραμμικών δυνάμεων του φιλμ λιπαντικού των εδράνων αντιμετωπίζεται σε αυτή τη διατριβή υπολογίζοντας κατευθείαν τις μη γραμμικές δυνάμεις που ασκούνται από το φιλμ στον άξονα. Η παραδοχή αυτή βοηθάει την προσομοίωση ιδιαίτερα σε περιπτώσεις που δεν υπάρχει σημείο ισορροπίας λόγω περιδήνησης μεγάλου εύρους ή λειτουργείας σε κρίσιμη ταχύτητα. Επίσης, για λόγους γενικότητας και πληρότητας της προσομοίωσης, δεν γίνεται η κατά κόρων κατά τη βιβλιογραφία παραδοχή του εδράνου απεριόριστου μήκους (infinite long bearing) ή του εδράνου αμελητέου μήκους (infinite short bearing). Αυτές οι δύο ακραίες υποθέσεις για το έδρανο επιτρέπουν αναλυτικές εκφράσεις για τη υδροδυναμική λίπανση αλλά δεν απαντώνται απαραίτητα στην πραγματικότητα. Στην παρούσα διατριβή χρησιμοποιούνται πεπερασμένα έδρανα τα οποία επιλύονται με ήδη γνωστό και αξιόπιστο τρόπο, όπως πολλοί ερευνητές έχουν προτείνει, χρησιμοποιώντας τη μέθοδο των πεπερασμένων διαφορών.

Η ανάγκη για την παρουσία εσωτερικής (υστερητικής) απόσβεσης στην προσομοίωση του συνεχούς άξονα είναι αναπόφευκτη, από τη στιγμή που απαιτούνται λύσεις πάνω στο συντονισμό, ούτως ώστε να αναδειχθούν οι επιδράσεις των βλαβών, που στη σειρά του συντονισμού γίνονται εντονότερες. Ο απειρισμός της απόκρισης, απουσία εσωτερικής απόσβεσης, δεν αφήνει περιθώρια για διερεύνηση των επιπτώσεων των βλαβών πάνω στην κατάσταση συντονισμού και για το λόγο αυτό η εσωτερική υστερητική απόσβεση ενσωματώνεται προκειμένου να επιτρέψει υπολογισμό της απόκρισης. Ο τρόπος με τον οποίο εισάγεται η υστερητική απόσβεση δεν διαφημίζεται από την βιβλιογραφία καθώς η απόσβεση εισάγεται με τη χρήση του μιγαδικού μέτρου ελαστικότητας και διάτμησης αλλά η επίλυση του προβλήματος αντιμετωπίζεται με τη χρήση μόνο πραγματικών αριθμών προκειμένου να είναι εφικτή η εισαγωγή των εδράνων στο σύστημα.

Διεξάγεται μια εκτεταμένη ανάλυση των χρονοσειρών του συστήματος με και χωρίς βλάβη, προκειμένου να διερευνηθούν τα συμπτώματα κάθε βλάβης στη σειρά χρόνου και συχνότητας. Η ανάλυση χρόνου-συχνότητας εκτελείται χρησιμοποιώντας το Συνεχή Μετασχηματισμό Wavelets (CWT) στις πειραματικές και αναλυτικές χρονοσειρές προκειμένου να εξαχθεί το μεταβλητό φαινόμενο συζευξίων που οφείλεται αποκλειστικά στην ανοιγκλείνουσα ρωγμή, από τους άλλους δύο κύριους λόγους της σύζευξης, δηλ. αυτούς των ανισόπροσωπων εδράνων και των συζευγμένων εξισώσεων του περιστρεφόμενου άξονα.

Εδώ χρησιμοποιήθηκε η ιδέα της χρήσης εξωτερικού ηλεκτρομαγνητικού διεγέρησης οριζόντιας διεύθυνσης, κατάλληλης συχνότητας και εύρους, ώστε να αναδειχθούν χαρακτηριστικά των βλαβών κατά τη λειτουργία. Η κατακόρυφη απόκριση, εξ αιτίας της σύζευξης ταλαντώσεων λόγω της ρωγμής, ενισχύεται όταν η ρωγμή βρίσκεται σε θέση όπου οι ενδοτικότητες σύζευξης γίνονται μεγαλύτερες, σε σχέση με την οριζόντια διεύθυνση

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εφαρμογής της ηλεκτρομαγνητικής διέγερσης στον άξονα. Αυτή η μεταβλητή σύζευξη παρουσιάζεται μόνο λόγω της ρωγμής και χρησιμοποιείται τελικά για την ανίχνευση της. Η εξωτερική διέγερση χρησιμοποιείται επίσης και στην περίπτωση της ανίχνευσης φθοράς, δεδομένου ότι η ανάλυση των ταλαντώσεων στο πεδίο του χρόνου ή της συχνότητας, δείχνει ότι ενισχύονται συγκεκριμένες αρμονικές όταν η φθορά υπεισέρχεται στο σύστημα.

Και οι δύο μέθοδοι για την αντίστοιχη ανίχνευση των βλαβών δοκιμάζονται σε ένα πραγματικό πειραματικό σύστημα. Η φιλοσοφία της μεθόδου ανίχνευσης των βλαβών βασίζεται στο ότι και οι δύο βλάβες πρέπει να ανιχνευθούν κατά τη λειτουργία του συστήματος. Πράγματι, υπάρχει η ανάγκη για έγκαιρη διάγνωση των βλαβών, σε πρώιμο στάδιο τους, στο χρονικό διάστημα που μεσολαβεί ανάμεσα σε δύο διαδοχικές συντηρήσεις διασφαλίζοντας την ασφαλή λειτουργία των μηχανών. Η έκταση των προς ανίχνευση βλαβών πρέπει να είναι τουλάχιστον 20% (της ακτίνας) για τη ρωγμή και 20% (της ακτινικής χάρης) για την φθορά ώστε η μέθοδος ανίχνευσης να χαρακτηρίζεται αποδοτική.

Οι κύριοι στόχοι της παρούσας διατριβής είναι: Η προσομοίωση ενός περιστρεφομένου και εσωτερικά αποσβενύμενου συνεχούς άξονα, εδραζόμενου σε φθαρμένες εδρές νηματώσεων. Η προσομοίωση και ο υπολογισμός των τοπικών και διακρατικών ενδοτικής συζεύξης της περιστρεφόμενης ανοιγοκλείουσας ρωγμής και της προσομοίωση των συζευγμένων ταλαντώσεων του ρηγματωμένου συστήματος άξονα-εδράνων. Η αναλυτική και πειραματική εφαρμογή του ρηγματωμένου και του φθαρμένου συστήματος και η διερεύνηση των επιδράσεων της ρωγμής και της φθοράς στην ταλαντωτική του συμπεριφορά. Η ανάπτυξη μεθόδων έγκαιρης ανίχνευσης της ρωγμής και της φθοράς.
Preface

This dissertation started at September 2004 in Machine Design Laboratory of Mechanical Engineering and Aeronautics Department in University of Patras under the supervision of Pr. Christos A. Papadopoulos. The subject is an extension to my undergraduate work during the fulfillment of diploma thesis in the same laboratory. During this thesis work many persons of different but familiar research interests helped the current work as colleagues or co-operators or as teachers with a special acknowledgment to them to be made in the sections of professional and personal acknowledgments.

It was many times that felt lucky to have some specific papers in front of me especially for these papers that guided the objective of this work. They are presented in the chapter of references among to many other papers that coexist as significant works for completeness in the state of the art.

In the first year of this work the main aim was to achieve a different simulation for variable crack compliance by geometric definition and compliance calculation of the crack as it is passes from tension to suppression. At the same time the coupling effect that a breathing crack produces was simulated by calculating the variable coupling local compliances during rotation. I would say that the first year was a period of fracture mechanics with the field of cracked rotor vibration not touched at all yet.

In the second year of this work the vibration of cracked structures exist in my simulations as long as in an experimental procedure with a cracked beam in order to evaluate the coupled response of the beam for all the range of crack shape during static rotation. At the same time the vibrations of cracked continue shafts become in the field with the simple assumption of linear bearing mounting in order to observe three different coupling reasons that appear in such systems: the crack, the bearing anisotropy and the coupled equations of motion.

In continue and during the third year the “rich” Rayleigh equation of motion for continuous rotors is combined with nonlinear fluid film bearings incorporating internal damping with the persuasion that a bit different simulation of rotor bearing systems can be achieved. The greater percentage of this period was dedicated in forming and solving a dynamic system of equations that simulate the rotor bearing system response. In the same time, and after the meeting and the start of cooperation with Dr. Pantelis Nikolakopoulos, an engineer from the industry, a new trend comes up and the subject of worn bearings is introduced in the field so as to combine the up to the moment model in the worn bearings in order to see if there is a
point on worn bearing effects in the rotor response, a matter that is much less investigated relatively to the crack effects in rotors.

During the last year some observations about the effects of worn bearings in rotors response give a point in bearing wear detection through time-frequency analysis. Additionally the widely researched, mainly from Pr. C. A. Papadopoulos during last 20 years, coupled response due to crack is used in this work in order to develop a method using external excitation able to isolate and extract the dynamic crack coupling during steady state operation of an experimental machine constructed for this reason. The use of the continuous wavelet transform judged efficient in extracting this rapid in time coupled response due to crack and also revealed specific harmonic amplification in the worn bearing system when it passes through resonance. Some of the results exist just for validating reasons while others become the tool of the current identification matter.

The entire simulation codes were developed in the software of Visual Basic® and Mathematica®, while signal processing and signal decompositions were performed in Matlab®. For the experimental data acquisition the Labview® software with the National Instruments® hardware were used.

Dipl. –Eng. Chasalevris Athanasios

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Introduction

I.1. The general problem and the significance

Transverse vibrations in high speed shafts in modern machines, principally turbomachines, under the coexistence of a transverse crack and of wear in fluid film bearings are the subject of the present work. A shaft is regarded as a high speed shaft if it rotates at a speed above critical, i.e. a speed at which transverse vibrations of considerable magnitude arise. On the basis of this definition, the shafts in modern turbo machines are for the most part high speed shafts. For this reason, an analysis of their operation touches on problems of oscillatory motion in the critical and post critical speed ranges, the determination of frequency ranges for natural vibrations and critical speeds, and the assessment of stability, particularly stability in the post critical range.

Additionally to the rotor bearing system concept, important achievements have been made during last 50 years in order the knowledge of the dynamical behavior of cracked shafts to allow to recognize the presence of a crack and to stop the cracked shafts in time before catastrophic failures. Close inspections revealed that in many shafts the crack had already propagated up to a depth of almost 50% of the diameter, which is obviously a very critical situation. Bearing in mind the fact that it is generally believed that propagating velocity increases exponentially, in many of these cases some days of operation would have been sufficient to provoke a catastrophic failure, with the loss of the complete machine train and with high risks for people and other equipments. In some cases instead the machines burst, generating catastrophic failures with heavy damages to the plants. Very high costs were involved in the reconstructing of the machines and the complete plant, as well as in the lost production for several years. When a crack is discovered in time, the cracked rotor can be substituted by a spare rotor in a few days or weeks, with affordable economic losses. This situation explains the increasing interest in behavior of cracks in general and of cracked shafts in particular. Starting from the 1980s up to the present, researchers from everywhere in the world has contributed with papers addressing different topics related to cracks in rotating shafts. The total number of papers is probably now close to 1000, and is still increasing. Anyhow, only very few papers present experimental results and generally these are related to very simple test-rings, like the Jeffcott rotor. Seldom can these results be extended to real rotors.

Failures associated with bearings and lubrication systems are one of the leading causes of forced outages in turbine generators. According to statistics compiled by the Edison Electrical Institute (EEI), bearing failures were responsible for 23% of all the forced outages for turbine generators rated at 600MW and over for the period 1967-1976. A similar Electric Power Research Institute (EPRI) study for fossil units of 600MW and over found that during the period 1964 to 1973, bearing and lubrication system failures were responsible for a “higher
rate of full force outage than any other turbine generator problem”. Analysis of these data indicates that problems associated with turbine generator bearing and lubrication supply systems costs the utility industry over $185M annually in replacement power costs. These conclusions are supported by findings from workshops held by EPRI as well as by statistics compiled by the National Electric Reliability Council (NERC) (U.S.). Relatively few of these turbine generator bearing failures occur during normal operation. Except of cases of disruption of the lubricant supply, the lubricant film thickness at full speed is sufficient to prevent damage to the bearings. Rather, the problems occur at low speed operation where the film thickness thin sufficiently to allow wear by fine contaminant particles and direct metallic contact. A recent study of the root causes of turbine generator bearing failures indicated that 54% of all bearing failures were attributed to contamination of lubricating oil. Since over half of all turbine generators forced outage hours are attributable to contamination in the lubricating oil system, further improvements to minimize contamination and wear related problems in turbine generator lubricating oil systems are required. Many studies have been performed on the transition from hydrodynamic to boundary lubrication at speeds offering the significant knowledge of the criteria for hydrodynamic lubrication. Also, studies for quantifying the progress or to determine the effect of wear on the hydrodynamic lubrication have been made during last 20 years but these studies stand away from the development of criteria for wear diagnosis.

Since the significance of developing the criteria for crack and wear diagnosis is in a brief manner unfolded, a detailed presentation of the present situation of rotor bearing system simulation, of crack effects in rotors and of wear bearing effects in rotor dynamics is performed in continue, while in the end of this introduction there is the detailed methodology of this work and the points in which the different point of view can be observed.
I.2. Historical Review and Present Situation

I.2.1 Continuous Rotor Systems

I.2.1.1 About the Development of Motion Equations

Most of the existing extensive literature on vibrations in flexible shafts up to early 60’s was concerned with determining the frequency of natural vibrations and critical speeds of the shafts. This trend in research was formerly linked with the solution of a basic problem, that of resonance elimination. This problem arose out of the requirements peculiar to a definite level of technical development, and its solution at this level was sufficient to ensure reliability. However, modern production of high powered steam and gas turbines and turbo generators with extremely flexible shafts rotating at speeds greatly in excess of critical speed and the construction and use of other high speed machines create problems of maintaining strength and reliability; the solution of these problems makes the study of vibratory motion necessary.

The presence of various sources of excitation, producing vibrations of varying intensity and frequency, and the effect of the energy dissemination factor required an analysis which would interconnect the operating loads (including friction forces) and the vibratory process from one hand and the shaft stresses on the other. Examination of existing data up to early 60’s showed that there has been insufficient development of the theory of vibrations in its application to rotating shafts. It is, therefore, not surprising that designers and stress men always encountered difficulties when designed shafts for high powered turbo machines, due to the confusion of ideas on the possible behavior of shafts in individual cases. In particular the following questions constantly arose. What is the nature of the transitional process through critical speed? Which loads produce constant flexure, and under which loads the shaft rotate "within" the elastic line? How do the mean and variable stress components alter under various operating conditions, and what stresses arise in the shaft when there are elastic vibrations in the bearings? How does a shaft drive behave when a round section shaft is connected to a shaft with a section in which the principal moments of inertia are dissimilar? These, and a series of other problems, where always present. The position with regard to friction effects in rotating shafts was even less clear, if allowance is made for the fact the rotation factor creates special conditions for the operation of friction forces. The confused state of many questions prevents correct analysis of experimental results obtained from research on shafts under actual operating conditions.
The “Flexible Shaft Theory” made its appearance in the 1870’s in connection with turbine techniques which were novel at that time. The principal trend in the development of this theory at the end of 19th century and the beginning of 20th century was towards the determination of critical speeds for shafts with concentrated and dispersed masses and with variable sessions; in essence, this was equivalent to the development of the theory of transverse vibrations in shafts. Research on specific problems connected with shaft vibration, initiated by Rankine in [1] 1869, and Laval in 1889, was continued by the turbine expert A. Stodola, who made theoretical and experimental studies of many fundamental phenomena (the gyroscopic effect of disks mounted on the shaft, ‘secondary resonance phenomena’, the stability of a shaft with one disk, etc.). The results of these investigations were collected in his principal work in 1920’s. The critical speeds of shafts with distributed mass were also examined by Grammel. In the same period Kimball, Newkirk and Robertson studied certain problems of vibratory motion and stability. Kimball and Newkirk was the first to show (in an elementary form) the possibility of shaft instability in the post critical range due to friction forces; Robertson attempted to give an elementary explanation of the practical possibility of stability in the post critical range. The Soviet scientist E. L. Nikolai examined the stability of transverse and torsion vibrations in a shaft with a disc mounted in the centre, and the stability of a shaft with a disc attached to the free end. P. L. Kapitsa pointed out that a flexible shaft could become unstable due to friction conditions in its sliding bearings.

Since the war, a number of works had been published which dealt with the determination of frequencies of natural vibrations in rotating shafts, applicable to the cases in which these frequencies depend on the speed of rotation. This is a more general problem than the determination of critical speeds, because the latter are special frequency values of natural vibrations, which are equal to the corresponding shaft speeds.

This general approach was made necessary by the extension of work on disturbing forces; the study of this subject was previously restricted to unbalancing forces. V. Y. Natanzon in and the design offices of certain factories made important contributions to this development. Foreign literature on this subject includes works by Bodganoff, Green and Föppl.

Additionally, at about early 60’s individual work was published related directly with the analysis of vibratory motion; examples of such work are of V. Y. Natanzon, V. A. Grobov, of J. Dick and of certain others.

Thus, the wide range of literature on vibrations of rotating shafts up to 60’s (only the most important works are given) provides no exhaustive analysis of problems relating strictly to vibratory motion; in addition, the published up to that time works provide no final solution for a number of problems relating to the determination of natural vibration frequencies and stability.
The monumental work of F. M. Dimentberg, “Flexural Vibrations of Rotating Shafts”, in [2] comes in 1961 to give results into the basic problems in the transverse vibration theory which relates directly the design and the calculation of strength in shafts of turbo machines. The work extended to the determination of vibration frequency and critical speeds, and to problems of natural vibration in connection with external forces and friction forces. A qualitative analysis of phenomena had been used in an attempt to clarify problems which had not up to that time been analyzed and elucidated, so that designers to be able to make the correct assessment of shaft operation. A number of the hypotheses in this work had been confirmed by experiments carried out at the “Elektrosila” works, and the Leningrad Foundry. Valuable contribution was made by the collaboration in setting up experiments and using the results obtained for analysis of theoretical problems.

In the same period and about 1965 an also monumental work of A. Tondl, “Some Problems of Rotor Dynamics” in [3] is based in theoretical and experimental work carried out by the author at the National Research Institute of Heat Engineering in Prague over a period of 10 years. The purpose of this research was to detect the causes of excessive rotor vibration and to provide data for the design of rotating machinery. The theoretical analysis was complemented by experimental investigations which served not only to verify and amplify theoretical results but, in many cases, to provide the stimulus for further theoretical investigations.

S. Timoshenko at about 1954 added the effect of transverse shear to the Rayleigh equation, which already incorporated effect of rotatory inertia, forming the famous “Timoshenko Beam Equation”. Both effects tend to lower the natural frequencies.

In studying the effects of constant axial torque on the critical speeds of a slender shaft, Golomb and Rosenberg found that it’s critical speed always decrease with axial torque. The influence of gyroscopic effects on the critical speeds of rotor systems already studied by Green and Dimentberg in [2] were investigated further also by Eshleman and Eubanks in [4] at about 1967.

The significant contributions to the rotor dynamic literature at about 20 years after war cannot leave out the independent and conscientious work of Toshio Yamamoto in his Nagoya University Laboratory on numerous topics related to rotor dynamics. His work focused on fundamental concepts related to the dynamics of high speed machinery and included meticulous laboratory test rigs to back up his analytical predictions. All of his work was done without the benefit of a high speed digital computer or sophisticated electronic test equipment. Some of his work was documented in 1954 and 1957 with the publication of two Nagoya University Memoirs entitled “On the Critical Speed of a Shaft” and “On the Vibrations of a Rotating Shaft”.

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I.2.1.2 The Role of Damping

In continue and becoming in earlier days, from late 1960’s and on, the rotor models and the fluid film bearings start to be researched in combination since the matter of damping, internal viscous and internal hysteretic, in vibration behavior is a significant point in turbo machinery design. It was earlier when the internal damping became a research object since its effect in critical speeds and stability is strong. Various works incorporate the continuous rotor equations, including the effects of transverse shear, rotatory inertia, and gyroscopic moments with fluid film bearings that are considered as short or long. Vibration analysis of rotating shafts with boundary conditions for long and short bearings is performed in late 60’s from Eshleman and Eubanks in [4] in order to facilitate the study of the influence of the axial torque on a torque-transmitting rotor’s critical speeds, comparing the results to classical results of Euler-Bernoulli and Timoshenko theory to determine the relative importance of the rotor’s “secondary phenomena” in a critical speed calculation.

It is however, well known that bearings have flexibility which inherently lower the critical speeds but, at least with fluid film bearings, shop tests normally confirmed the results of the rigid bearing calculations. The primary cause of this of this apparent discrepancy is the substantial bearing damping which acts in series with shaft flexibility, thereby contributing to a stiffening of the bearing. The effect which depends on the ratio between shaft and bearing stiffness is included by a method proposed by Lund in [5] in 1974. Lund determines the damped natural frequencies of a general rotor in fluid film journal bearings and calculates the threshold speed of instability and the damped critical speeds. The bearings were supposed to have linearized stiffness and damping coefficients and the calculation includes hysteretic internal damping in the shaft, internal viscous damping in the bearings and destabilizing aerodynamic forces showing the results for an industrial multistage compressor.

As the operating speed of a rotor enters the supercritical region, the destabilizing effect of internal damping becomes increasingly important. Material damping and internal friction are examples of internal damping. Internal damping is represented by two components: viscous and hysteretic. The main difference between the two types of damping is that the viscous damping is frequency depended, whereas the hysteretic damping is not. Internal viscous damping has been well researched, both analytically and experimentally, illustrating stability thresholds as well as whirl speed determination. Damped natural frequency for hysteretically damped spinning shafts, however, has only been solved numerically using the finite element method. With the increasing demand for high speed turbo machinery applications, analytical solutions of internally damped Timoshenko shafts leaded to more ideal rotor design due to the easy implementation of damping.
Introduction

The internal damping in spinning shafts had been investigated by Kimball in [6] for the first time, but Dimentberg in [2] and Gunter in [7] illustrated how above the critical speed, internal viscous damping leads to instability in the rotating shaft. As well Dimentberg and Gunter formulated analytical solutions for the internal viscous damping forces. Dimentberg went on to show that hysteretic damping, unlike internal viscous damping, is destabilizing at all speeds. Ehrich in [8] studied the stability relationship between internal damping and external damping and noted that the rotational speed at which the rotor becomes unstable is governed by the ratio of internal and external damping. The addition of external damping can raise the threshold at which the motion will become unstable. Vance and Lee in [9] investigated the stability of high speed rotors with internal friction. Their model was that of a single unbalance disk mounted on a flexible shaft, modeling the internal friction as viscous damping. The stability was evaluated using mathematical methods and verified by numerical solutions of the governing differential equations. The results agree with Gunter and Dimentberg in that instability occurs at running speeds above the first critical speed. Vance and Lee also demonstrated how the stability threshold can be raised by adding external damping to the system. Lund in [10] developed a method for calculating the threshold for instability as well as the damped natural frequencies of a flexible rotor in fluid-film bearings. In his calculations, Lund represented the rotor by a series of lumped masses connected by uniform shaft sections. With this representation, Lund stated that any practical shaft geometry can be accommodated. Lund also incorporated hysteretic damping into his model. Crandall in [11] developed equations of motion for internally viscous damped systems. He also gave a physical explanation of the destabilizing effects of internal damping.

Over the past twenty years, numerous studies on internally damped rotating shafts have been carried out utilizing the finite element method. Zorzi and Nelson in [12] performed finite element simulations on a rotor dynamic system which included internal viscous damping and hysteretic damping. As well, Zorzi and Nelson concluded that internal viscous damping had a destabilizing effect above the first critical speed while hysteretic damping had a destabilizing effect at all speeds. Zorzi and Nelson's research was lacking in that they neglected the effects of rotary inertia and shear deformation. Ozguven and Ozkan in [13] did however take Zorzi and Nelson's work further by including Timoshenko beam theory in their solution. Ozguven and Ozkan calculated the complex eigenvalues for a Timoshenko shaft and compared the results to that of a Rayleigh beam. Hashish and Sankar in [14] developed explicit expressions for the virtual work done by the internal damping forces and included them in their finite element model. Gao and Zhu [15] developed finite element equations for a Timoshenko spinning shaft with both internal viscous and hysteretic damping in complex form. Gao and Zhu in [16] stated that the complex representation results in reducing the computational time.
by a factor of four compared to that of the real form representation. Chen and Ku in [17]
developed a three-node Timoshenko beam finite element model to study whirl speeds of
rotor-bearing systems with internal viscous and hysteretic damping. The advantage of this
new finite element model is that basic convergence requirements are automatically satisfied.
The results were compared to previous works using finite element methods.

Very recently, during 2004, and in order to make the role of internal damping clearer, Genta in
[18] provides some incorrect statements on the effect of hysteretic damping. Genta aims at
clarifying the issue of internal damping in terms of the practical applications of the theory in an
unequivocal way. Genta declares that all damping which can be associated with the no
rotating parts of the machine has the usual stabilizing role as in structural dynamics, but
damping of rotating elements can trigger instability in the supercritical range. What actually
happens is that rotating damping couples rotational motion and vibration, causing energy to
be transferred from the former to the latter. This phenomenon is general, occurring every time
energy dissipation takes place in a rotating system: eddy current dampers in which energy is
dissipated in the rotor can cause instability in the supercritical regime, although energy
dissipated into the stator does not. From the viewpoint of stability, there is no difference in the
qualitative effect, whether damping is modeled as viscous or as hysteretic damping. However,
from time to time the statement that, while viscous rotating damping is stabilizing in sub
critical conditions and destabilizing in supercritical conditions, hysteretic damping is always
destabilizing at any speed, can be found even on papers published in well known journals. As
an example, the conclusions of a recent ASME paper [19] contain the explicit, as he refers,
statement: Hysteretic damping is destabilizing at all speeds. Often this statement is
substantiated by quotations from the book by Dimentberg in [2] and the paper by Lund in [20].
In the first case it is a misquotation: speaking of hysteretic damping ~page 28 of the English
edition; Dimentberg wrote “consequently...the shaft will be unstable for any speed of
rotation,” but then adds, as an obvious comment “above the critical, of course.” This
sentence is often quoted incompletely and its meaning is deeply changed. In the second case
the incorrect statement is actually present in [20]. Although this error was pointed out by
various authors ~included the present (Genta) and also made clear by Crandall in [21], this
idea propagated in a subsequent paper [12], also quoted in [19].

I.2.1.3 The Earlier Days Rotor Bearing Systems – Simulations – Solution
Methodologies

Special attention to the effects of various nonlinear mechanisms on the dynamic behavior of
rotor bearing systems has been paid from 1975 and on from Yukio Ishida. Ishida has also
extensively investigated the use of modern digital signal processing as a valuable tool for the
analytical and experimental investigation of vibrations in rotor dynamic systems. Several topics associated with more recent studies are included in his very collective and recent work “Linear and Linear Rotor dynamics” in [22].

A significant contribution in analytical solutions to discrete or continuous rotor systems, being stimulated from acquaintance with the work of F. M. Dimentberg, regarding individual various works of past 3 decades, is made by C. W. Lee in a recent work “Vibration Analysis of Rotors” in [23]. In previously published rotor dynamics texts, the behavior of simple rotors had been of a primary concern, while more realistic, multi degree of freedom or continuous systems are seldom treated in a rigorous way, mostly due to the difficulty of a mathematical treatment of such complicated systems. When one wanted to gain a deep insight into dynamic phenomena of complicated rotor systems, one had, in the past, either to rely on computational techniques, such as the transfer matrix and finite element methods, or cautiously to extend ideas learned from simple rotors whose analytical solutions were readily available. The former methods are limited in the interpretation of results, since the calculations related only to the simulated case, not to a more general system behavior. Ideas learned from simple rotors can, fortunately, often be extended to many practical rotor systems, but there is of course no guarantee of their validity.

In the treatment of rotor-bearing system dynamic phenomena, the need recently has been raised for analysis of the highly nonlinear forces of fluid film bearings in combination with a continuous rotor model. In cases of large amplitude vibrations, which approach the radial clearance in bearing locations, linear theory has been judged as insufficient since prediction for journal vibrations yields amplitudes larger than the radial clearances. This large vibration amplitude is not always a result of high unbalance excitation nor is it generally due to external loading, but it can be produced if the rotor bearing system functions under resonance. In such cases, the shearing forces grow to large magnitudes, and this has a direct effect on the hydrodynamic lubrication inside the bearings. Since the shearing force magnitude is significant in the continuous shaft model (including torsion and gyroscopic moments), it should be included to aid in an accurate modeling of the shaft’s dynamic properties. The extension of this correlation can be a bearing hydrodynamic function that accounts for the dynamic properties of the entire shaft instead of the journal mobility. The expression of the journal dynamic properties as a function of the entire shaft vibration results in precise journal mobility, especially when defects such as cracks are present in the shaft.

The above consideration has been treated by many researchers with the various combination of a shaft model with fluid film bearings. Many methods for transverse vibration analysis of rotor systems have been developed, and these may be divided into two major classes. The
first contains discretization methods, such as FEM and the Transfer Matrix Method, in which the rotor system is approximated by a finite degree-of-freedom system whose motions are described by ordinary differential equations [10,14,24,25]. The second includes the analytical method in which a rotor system is treated as a distributed parameter system whose motions are described by partial differential equations [2,4,26]. C.-W. Lee et al. [27] applied modal analysis to a continuous rotor system with various boundary conditions (isotropic and anisotropic); whirl speed and mode shapes (forward and backward) are obtained as spin speed and boundary conditions vary. The effects of asymmetry in boundary conditions on the system dynamic characteristics are also investigated. Also, C. W. Lee et al. [26] use a modal analysis technique to investigate the forced response analysis of an undamped distributed parameter rotating shaft. A study of the resulting non-self-adjoint eigenvalue problem is presented.

Modal analysis of an asymmetrical rotor-bearing system, which consists of asymmetrical Rayleigh shafts, asymmetrical rigid disks, and isotropic bearings, is performed by Jei et al. [28]. In [29,30], Adams models a rotor as a separate system with use of finite elements. The central feature of his analysis is a proper handling of various highly non-linear effects of journal bearings, which dominate the dynamic phenomena encountered during large amplitude rotor-bearing vibrations. The Transfer Matrix Method is used in calculating the transient response of a large-scale rotor bearing system using strong nonlinear elements in a study by Gu et al. [30], who obtain the transfer matrix via the Newmark formulation and determine the dynamic characteristics by integration.

A very recent work of Shen et al. [31] presents a fast and accurate model for the calculation of the bearing fluid film forces using free boundary theory and the variation method. The model is applied to the nonlinear dynamic behavior analysis of a rigid rotor mounted on elliptical bearings. The investigation shows that the rotor motion can be synchronous, sub harmonic, quasi-periodic, or chaotic at different rotor spin speeds through the use of Poincaré maps, bifurcation diagrams, and frequency spectra as diagnostic tools. The chaotic behavior can be presented in such nonlinear fluid film force assumptions, which often yield unexpected phenomena that are extremely sensitive to initial conditions, as Brown et al. and Chen et al. have demonstrated in [32] and [33], respectively. Multi-span Timoshenko beams connected or supported by resilient joints with damping is used by Hong et al. [34,35] in order to achieve exact solutions for a distributed parameter system. An analysis of whirl speeds for rotor-bearing systems supported on fluid film bearings is presented in [36] by Kalita et al., using FEM to present the observation that the additional frequencies around half the spin speeds cannot appear when using the short bearing approximation, but appear only with finite bearing dynamic coefficients. Ehrich [37] observes sub critical, super harmonic, and chaotic
responses in rotor dynamics. An emphasis on oil whip phenomena in the non-linear dynamic behavior of rotor–bearing systems is given in [38] by JianPing et al., using FEM for the rotor motion and the short bearing approximation for fluid film forces.

It is understandable that since rotating machines represent the largest and most important class of machinery used for fluid media transportation, for metal working and forming, for energy generation, for providing aircraft and marine propulsion as long as for many other purposes, a great amount of papers, books, patents, reports and various texts has been published during the last 100 years. It is a very difficult matter and surely out of the authors ability to recognize and describe the significance of each work as long as to include each contribution in the entire field of rotating shafts in this introduction. On the other hand a general description of the direction in which various researches made their contributions has been probably made during this brief introduction.

1.2.2 Crack Effects in Rotor dynamics

Cracked rotating shafts have been the object of studies and investigations since the 1960s. It seems that the first paper on cracked structures has been published in 1957; therefore we now have a story lasting almost 50 years. Important achievements have been made during these years; the knowledge of the dynamical behavior of cracked shafts has allowed to recognize the presence of a crack and to stop the cracked shafts in time before catastrophic failures. As it was referred in previous paragraph of “The general problem and the significance”, close inspections revealed that in many shafts the crack had already propagated up to a depth of almost 50% of the diameter, which is obviously a very critical situation. Very high costs were involved in the reconstructing of the machines and the complete plant, as well as in the lost production for several years. When a crack is discovered in time, the cracked rotor can be substituted by a spare rotor in a few days or weeks, with affordable economic losses. This situation explains the increasing interest in behavior of cracks in general and of cracked shafts in particular. Starting from the 1980s up to the present, researchers from everywhere in the worlds have contributed with papers addressing different topics related to cracks in rotating shafts. The total number of papers is probably now close to 1000, and is still increasing. Anyhow, only very few papers present experimental results and generally these are related to very simple test-rings, like the Jeffcott rotor. Seldom can these results be extended to real rotors. Given this situation, very recently a special issue of the journal Mechanical Systems and Signal Processing was dedicated to the state-of-the-art of modeling the dynamical behavior of cracked rotors, including the inverse problem of the identification of cracks in rotating machinery. Some authors among the very rich quantity of
different contributors were invited to present a specific topic in the wide field of all the different aspects of cracked rotors.

This special issue is parted from 6 papers, covering different aspects of cracked rotors such as, fracture mechanics approach in stiffness variations, main characteristics of the behavior of simple cracked rotor models, industrial machine case histories with insight to the frequency and danger crack effects, high accuracy approximation of effect of cracks of any shape, multiple crack simulation and crack detection, and other peculiar aspects such as accurate model of the breathing behavior influenced by possible thermal transients, helical development of cracks and nonlinear effects occurred to cracked industrial machines. In continue, these aspects are presented, in a brief way, in the corresponding paragraphs.

I.2.2.1 Industrial Machine Cracked Case Histories

Horizontal rotors are always imposed to periodic stresses and, therefore, a crack due to a fatigue is unavoidable. The proper diagnosis of machinery is necessary to prevent tragic accidents and the vibration monitoring is the most important tool for such a diagnosis system. In order to develop a monitoring system that can detect a crack in an early stage of propagation, it is important to know the vibration characteristics of a cracked rotor. A crack opens or closes due to the direction of the lateral deflection. Therefore, a cracked rotor has nonlinear spring characteristics of a piecewise linear type. In order to diagnose the vibration characteristics properly, it is essential to understand the behavior caused by the nonlinearity. These piecewise linear characteristics make a directional difference in stiffness and this difference rotates with the rotor. As the result, the coefficients of linear and nonlinear terms in restoring forces become time dependent. According to the physical characteristics, cracked rotors can be classified into a class of nonlinear parametrically excited system.

In a horizontal rotor where gravitational force works, the tension and compression work in the upper and lower sides of the rotor, respectively. Since these stresses change periodically as a rotor rotates, the rotor has high probability to be destroyed by a fatigue crack. In order to develop a vibration monitoring system that enables detecting a crack during the operation, vibration characteristics of cracked rotors have been investigated by many researchers.

The accidents due to cracks in turbine generators have been reported since 1950s but theoretical and numerical investigations on the vibration characteristics of cracked rotors started from 1970s. The overview of the past studies on this field until 1989 is reviewed by Wauer [39]. When a horizontal rotor has a transverse crack, the crack area opens or closes due to the self-weight bending as it rotates. This is called breathing and this makes the characteristics of a cracked rotor nonlinear. This mechanism can be considered when a
physical rotor model is constructed. In 1976, Gasch [40,41], Henry and Oka-Avae [42] considered this breathing nonlinear mechanism by using different flexibilities for open and closed condition and solved the equations of motion by an analog computer. Many researchers investigated vibration characteristics of cracked rotors in detail qualitatively and quantitatively using various kinds of physical models. Since it is considered that the essential characteristics can be understood more easily by a simple physical model, Y. Ishida clarified various kinds of resonance which may occur in cracked rotors using models composed of a massless elastic shaft and a disc mounted at the center of the shaft.

This paragraph introduces case histories of cracked rotors in industrial machines and presents nonlinear effects on various kinds of resonance in cracked rotors using a simple rotor model similar to Jeffcott rotor. Even though the characteristics of this model cannot be considered rigorously as Jeffcott rotor model, for simplicity, this model is called as “Jeffcott rotor model”. In this model stationary and no stationary phenomena are both investigated.

Turbine generators developed very rapidly during 1950s–1970s and their sizes increased from about 60MW to about 500MW in this period. Since many turbines are constructed without consideration of the experiences of the foregoing products sufficiently, turbine rotors suffered from cracks and many tragic accidents occurred in those days. Since 1950s, cracks have been found in turbine shafts or in turbine blades and some kind of tragic accidents which probably happened due to cracks have been reported. Generally, it is very difficult to specify the cause of the accidents after the rotor system was destroyed completely. In 1953, the 1800 rpm steam turbine at the Tanners Creek power station in USA suddenly went into vibration, and it was found that a segment of approximately 160° had broken out in a wheel due to a crack [43]. In March 1954, a 3600 rpm generator rotor in Arizona, USA, burst while being balanced in the factory [44]. Several cracks were found before bursting in this rotor. In September 1954, a generator rotor in Cromby, USA, burst while running at 3780 rpm [45]. In December 1954, the LP steam turbine in Ridgeland, USA, burst during a routine over-speed trip test [46]. In 1954, a generator rotor in Pittsburg, USA, burst during an overrun test at 3920 rpm [47]. In 1970, a LP turbine burst during a running test in Nagasaki, Japan [48]. More than 60 people died and injured in this tragic accident [49]. The investigation team concluded that the cause of this collapse is cracks due to a stress concentration at grinded holes.

There are many cases where cracks were found luckily before a disaster. Coyle and Watson reported that, in 1956–1957, the shafts of three large turbine rotors developed fatigue cracks under very low nominal gravitational bending stresses in Castle Donnington, UK. The fatigue cracks propagated over 75% of the shaft section in two cases, and the cracking started all around the circumference and propagated no deeper than 1/16 inch in the other case.
Yoshida in [50] reported four cases that cracks are found in four steam turbine rotors during 1970–1971 in Japan. He concluded that the cyclic thermal stress which appeared during the frequent start up and shut down made cracks. Jack and Paterson reported that three cracks were discovered in three LP shafts in 500MW turbines at Ferry bridge power station during 1972–1974. Muszynska in [51] reported that at least 28 cracked failures happened within the period 1970s. These incidents are a 40% transverse crack of 500MW turbine generator, a circumferential 30 in long crack of MP rotor of 60MW turbine generator, a 45% transverse crack of the main generator rotor of 660MW turbine generator and a crack extending around complete circumference of the MP rotor of 350MW turbine at main HP coupling end. Cracks of other kinds of machines are also reported. For example, a rotor crack which expanded of 1201 of the cross section of the shaft of a wind tunnel fan for a car testing was reported in 1990 in Japan in [52]. In many cases, the occurrence of a crack is noted by an abnormal increase of vibrations. However, the development of vibration diagnosis system, which was installed to monitor the turbines started to reveal the detailed symptoms of cracks. Dimarogonas and Papadopoulos reported a crack failure found in 300MW turbine generator in 1983 in Lavrion power plant in Greece in [53]. He reported vibration spectrum where characteristic peaks are admitted.

From 1970s, cracks were found in turbines of several atomic power plants. This led to a much more serious situation for the people. Ziebarth and Baumgartner reported the vibration histories after cracks occurred in the 1300MW turbine in Cumberland power plant in USA and in Wuergassen nuclear power plant in Germany. In the former, an indication appeared and started to increase before 3 days prior to the shutdown. A double-frequency excitation was noted upon the coast down. In the latter, the horizontal and vertical vibrations of the LP turbine started to increase from 6 days prior to the shutdown. A pump shaft in Crystal River nuclear power plant broke due to a fatigue crack in a groove in 1986 and also due to crack in another part in 1989. In 1990, a crack distributed 25% of the cross section was found in a generator rotor of the 935MW turbine generator of Darlington nuclear power plant. Sanderson in reported the characteristic of the data obtained by the monitoring system in detail. It shows that the overall vibration started to increase about 5 days before the shutdown and the amplitudes of the harmonic resonance and the double-frequency resonance increased due to the occurrence of a crack. It can be imagined that there are many other cases, which are not reported to the public.
I.2.2.2 Effects of Crack in Simple Rotor Models

Linear Torsional Vibrations of Cracked Shafts

Walker et al. [54] presented two research projects for obtaining an improved understanding of interactions between large steam turbine-generators and electrical transmission systems. Of particular current interest to the industry is the correlation of transmission system disturbances to the rate of fatigue life expenditure of turbine-generator shafts. The first project consists of development of a torsional fatigue methodology for turbine-generator shaft materials and configurations, based on a test program. The second project involves capturing and analyzing data from a population of torsional vibration monitors with the objective of developing industry guidelines for design, application, and operation of electric power generation and transmission systems.

Williams et al. [55] presented the results of analysis and torsional fatigue tests on 25.4mm diameter laboratory specimens. The results of these experiments were used to predict the fatigue life of large diameter shafts subjected to torsional transients. These predicted results were verified with 127mm diameter specimens subjected to simulated torsional transient load history tests.

Stability analysis of Cracked Shafts

Stability charts are presented by Gasch [40] and by Papadopoulos and Dimarogonas [49] (Figure 1) where the stability regions of a shaft with a transverse breathing crack are presented for the coupled longitudinal and bending vibration mode, and by Huang et al. [50] for rotating shafts. The second chart of Huang shows the effect of the damping on the stability of the rotating system indicating that damping has a stabilizing effect on the cracked shaft vibrations. Yang reported that chaos due to nonlinearity induced by the crack may possibly occur [56]. Several works deal with the stability of cracked structures [57-60].

Transient Response of Cracked Shafts

When a cracked shaft passes through its critical speed, it presents a particular behavior. In fact, the transient vibration response develops oscillations that become violent inside a range of angular velocities near the critical speed [61,62]. Monitoring the transient response during passage through the critical speed could give an indication of crack existence. For certain eccentricities angles, during the passage, while accelerating or decelerating, the crack remains open until shortly after the maximum response is attained and closes after [63].
Figure 1 Stability charts depending on the crack depth and the vibration frequency. (a) Huang et al. [50] without damping, (b) Huang et al. [50] with damping, (c) Gasch [40] and (d) Papadopoulos–Dimarogonas [59].

A consistent Vibration Theory of Cracked Bars and Beams

A consistent continuous cracked bar and beam vibration theory was developed by Chondros et al. [64-70]. Particularly, Chondros modified the Christides and Barr [71] continuous cracked beam theory by taking into account the stress and displacement fields around the crack for modification of the stress and displacement fields throughout the bar and the beam. The Hu–Washizu–Barr variation formulation was used to develop the differential equation and the boundary conditions of the cracked rod or beam. The same continuous method was applied by Carneiro and Inman [72] for a cracked Timoshenko beam.

Damped Vibrations of Cracked Shafts with Thermal Effects

The material damping in a cracked structure increases due to the reciprocity of the temperature rise and strain. This increase is correlated to the vibration mode and the magnitude of the existing crack in the structure by Panteliou et al. [73]. The analytical
determination of the dynamic characteristics of the cracked structure yielded the damping factor of the bar, the material damping factor, and a good correlation of crack depth with the damping factor. Experimental results on cracked bars are in good correlation with the analysis. Similar results are presented by Zhang and Testa in [74].

**Vibrations of Cracked Structures in Viscous Liquid**

When a cracked shaft is rotating in a viscous fluid, then there is a change in the critical speeds and the amplitudes of vibration. Behera et al. [75] and Parhi and Behera [76,77] analyzed the effect of the fluid using the Navier–Stokes equations. The damping effect and virtual mass effect are also taken into account through the Navier–Stokes equation.

Gounaris et al. [78] applied the fail-safe criterion to a floating, ship-like, Timoshenko beam moving forward in waves, including both hydrodynamic and structural damping. The nominal stress near the crack, needed as input to the Paris equation for the evaluation of the cycles required to failure, is computed assuming known values for the SIF. The resulting fail-safe diagrams indicate the effect of various parameters of the problem.

**I.2.2.3 Crack Models**

Since that time plenty of papers were published dealing with crack models, the coupled transversal, torsional and longitudinal vibrations of cracked rotors and—of course—early crack detection to avoid catastrophic failure.

**A Simple Crack Model**

The deflection line of a shaft with a crack in the tension zone (Figure 2) is superimposed from two parts: the deflection line of the uncracked shaft and an additional deflection from the local flexibility of the crack. This additional part cannot be found from the bending theory. Because for the beam theory a crack is a weakening of the bending stiffness EI on a length zero, only a three-dimensional consideration (or a two-dimensional approximation) is able to yield this additional weakening of the stiffness [79].

In fact, due to the loss of symmetry a cracked round shaft produces a coupling of lateral vibration, axial and torsional vibration. However for the sake of simplicity we ignore these effects and focus our interest only on the lateral vibration. In [79] we find the crack flexibilities for a general cracked beam element with 6 degrees of freedom at each end. We simplify and pick up only the main flexibility $H_{22}$ and the cross-flexibility $H_{33}$ (Figure 3). These flexibilities are
presented in a dimensionless form. To get the physical flexibilities they have to be divided by $ER^3$, where $E$ is the Young’s modulus and $R$ the radius of the shaft.

![Figure 2 Deflection line; contributions of the uncracked shaft and the local crack flexibility without crack contribution.](image)

Figure 2 shows these flexibilities versus crack depth $a/R$. For small cracks ($a/R < 0.5$) the cross flexibility $H_{22}$ is much smaller than the main flexibility $H_{33}$, so going ahead the cross-flexibility is ignored completely. This has the advantage that the crack flexibility is represented by only one parameter. So the understanding of the analytical results will be eased.

![Figure 3 Main- and cross-flexibility of the open crack from [75].](image)

Thus the deflection at the position of the disc with an open crack nearby can be written as
Where $h_0$ is the flexibility of the round uncracked shaft and

$$\Delta h_i = \frac{H_{22} f^2}{E R^3 l_6}$$

is the (translator) additional flexibility of the crack. Hereby the span of the shaft is $l$.

Turning the shaft with the crack into the pressure zone the crack will be closed due to the pre-stressing of the weight and $\Delta h_i$ will become zero; the shaft is round again. This behavior is represented by the hinge model shown in Figure 4.

---

**Figure 4 Crack model (hinge model) for the “breathing” crack.**

The hinge opens in the tension zone and closes in the pressure zone. Thus the crack mechanism is non-linear, respectively, bi-linear due to our assumptions. So for our further calculations we re-write in the following form:

$$\begin{bmatrix} w_{\zeta} \\ v_{\eta} \end{bmatrix} = \begin{bmatrix} h_0 & \Delta h_i \\ h_0 & 0 \end{bmatrix} \begin{bmatrix} f_{\zeta} \\ f_{\eta} \end{bmatrix},$$

Where $f(t)$ is the steering function being

$$f(t) = 0, \text{ when the crack is in the pressure zone and }$$

$$f(t) = 1, \text{ when the crack is in the tension zone.}$$
Approaches Based on Fracture Mechanics

The strain energy release rate (SERR) theory, combined with Linear Fracture Mechanics and Rotor dynamics theories, has been widely used over the last three decades in order to calculate the compliance that causes a transverse surface crack in a rotating shaft. In this section, the basic theory of this approach is presented, along with some extensions and limitations of its usage. The SERR theory is applied to a rotating crack and gives good results. The linear or nonlinear cracked rotor behavior depends on the mechanism of opening and closure of the crack during the shaft rotation. A brief history of the SERR theory is presented. In the 1970s, this theory met with rotor dynamics as a result of research conducted on the causes of rotor failures in power industries. The main goal of this research was to give the engineer an early warning about the cracked situation of the rotor, in other words, to make the identification of the crack possible. Different methods of crack identification are presented here as well as those for multi-crack identification.

During the last three decades, great attention has been paid by several research scientists to the diagnosis of cracks in rotating machinery. The excellent review papers by Dimarogonas [80], Wauer [39], and Gasch [40] cover many aspects of this area and give valuable information and knowledge. The challenge of modeling a crack is one of the most significant issues in this area. The theory of SERR combined with rotor dynamics in the early 1970s, when the detection of fatigue cracking became a necessity in power plants. Dimarogonas combined the fracture mechanics field with rotor dynamics and this concept led to calculation of the full local compliance matrix due to the crack. If the crack opens and closes in time, depending on the rotation and vibration amplitude, then the system is nonlinear. Coupled vibrations appear due to the crack. In addition to the second harmonic of rotation and the sub harmonic of the critical speed, two more families of harmonics are observed:

1. Higher harmonics of the rotating speed due to the nonlinearity of the closing crack, and
2. Longitudinal and torsional harmonics that are present in the start-up lateral vibration spectrum due to the coupling.

The Irwin method of local compliance computation has been extended for beams of five degrees of freedom (DOFs) by Dimarogonas and Paipetis [81] and for shafts and rotors of six DOFs by Papadopoulos and Dimarogonas, [82-84]. The additional displacement $u_i$, along the direction of the force $P_i$, due to the presence of a crack of depth $a$ in a beam of rectangular cross section, is calculated by applying Castigliano’s theorem and the equation of Paris [85]:

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\[ u_i = \frac{\partial}{\partial P_i} \int_{y_0}^{a} J(y) \, dy, \]

where \( J(y) \) is the SERR and \( P_i \) the corresponding load. \( J(y) \) depends on the crack depth and the applied generalized forces that are responsible for the different modes of fracture (opening, shearing or tearing). For general loading of the cracked cross section [85]:

\[ J(y) = \frac{1 - \nu^2}{E} \left[ \left( \sum_{i=1}^{6} K_{yi} \right)^2 + \left( \sum_{i=1}^{6} K_{yi} \right)^2 + (1 + \nu) \left( \sum_{i=1}^{6} K_{yi} \right)^2 \right] \text{ (plane strain)}, \]

where \( E \) is the Young modulus of elasticity, \( \nu \) the Poisson ratio, and \( K \) the SIFs for fracture modes I, II, III. The stress intensity functions depend, of course, on the cracked section geometry and on the applied loads.

The cracked circular cross section is considered to be divided into strips of width \( dx \) that are independent from each other, i.e. there are no tractions between two successive strips. This is not true near the ends of the crack tip. Bachschmid and Tanzi [86] used 3D FEM to show that, depending on the applied forces, there are no constant strains and stresses along the crack tip.

Finally, the elements of the compliance matrix are calculated by the following double integral:

\[ c_{ij} = \frac{\partial u_i}{\partial P_j} = \frac{\partial^2}{\partial P_i \partial P_j} \int_{-b}^{b} \int_{0}^{a} J(y) dy \, dx. \]

The calculation of the elements of the matrix is presented in [82-84] and [50,53,87,88] is here omitted. An extensive and detailed presentation of the local compliance calculation using the above-presented SERR theory is also presented by Theis in [79]. The full compliance matrix has the form:

\[ [C] = \begin{bmatrix} c_{11} & 0 & 0 & c_{14} & c_{15} & 0 \\ 0 & c_{22} & 0 & 0 & 0 & c_{26} \\ 0 & 0 & c_{33} & 0 & 0 & c_{36} \\ c_{41} & 0 & 0 & c_{44} & c_{45} & 0 \\ c_{51} & 0 & 0 & c_{54} & c_{55} & 0 \\ 0 & c_{62} & c_{63} & 0 & 0 & c_{66} \end{bmatrix}. \]

Modeling and Breathing Mechanism

When the shaft is rotating, then the crack opens and closes according to the stresses developed in the cracked surface. If these stresses are extensive, then the crack is open, resulting in reduced shaft stiffness; when the stresses are compressive, then the crack remains closed and has the same stiffness as the intact shaft. It could be also said that there is an intermediate situation between the above two (open and closed crack) that is partially opened or partially closed. This situation is presented during the transition from open to close
crack or vice versa. The crack opens and closes according to the shaft rotation in the case when the static deflection dominates the vibration of the rotating shaft. This is a very common situation in large turbine-generators rotors. Then, the compliance change, and consequently the shaft stiffness change, is a function of the angle of rotation or of \( \omega t \), where \( \omega \) is the angular velocity and \( t \) the time. The resulting equations of motion are linear with time dependent coefficients. In contrast, for light or vertical shafts where the static displacement is dominated by the vibration amplitude, the crack opening or closure is a function of this vibration and thus the equations describing the phenomenon are nonlinear.

When the crack is assumed to remain open during the revolution, then it follows the model of dissimilar moment of inertia presented by Dimentberg [2] and Tondl [3]. A simple nonlinear fatigue crack model that divides the load displacement diagram into three regions:

(a) Crack fully open,

(b) Crack partially open, and

(c) Crack fully closed, was presented by Cheng et al. in [89].

In the switching crack model the crack is assumed to open and close fully following a square pulse function (Figure 5). This behavior is connected to rather small cracks, while a deep crack could be associated with a harmonic variation [39,40,90-94].

![Figure 5 The closing behavior of (a) the hinge model (weight dominated) and (b) Mayes’ modified function \((1+\cos \omega t)/2\) for deep cracks [40].](image)

Mayes and Davies [93,94] considered a local reduction of the second moment of inertia in order to model the crack. This approach follows:

\[
\frac{\Delta I}{I_0} / (1 - \Delta I / I_0) = \frac{R}{c} (1 - v^2 f(x)),
\]
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where \( I_o, R, l, v, a \) and \( F(a) \) are the second moment of inertia, the shaft radius, the length of the section, the Poisson’s ratio, the non-dimensional crack depth, and the compliance functions varied with the non-dimensional crack depth \( a \).

In [59], the stiffness of the rotating shaft (Figure 6) was calculated by the formula:

\[
[K] = [K_3] + [K_1] \cos \omega t + [K_2] \cos 2\omega t + [K_3] \cos 3\omega t + [K_4] \cos 4\omega t.
\]

Sinou et al. [95,96] use the following function (Figure 7):

\[
f(t) = \frac{1}{2}(1 - \cos \omega t),
\]

whereas in [60,97,98], the breathing function was assumed to be the following cosine series, simulating the rectangular pulse (Figure 8):

\[
f(\omega t) = \frac{1}{2} + \frac{2}{\pi} \cos \omega t - \frac{2}{3\pi} \cos 3\omega t + \frac{2}{5\pi} \cos 5\omega t - \frac{2}{7\pi} \cos 7\omega t + \cdots.
\]

Darpe et al. in [99] proposed the concept of the closure crack line (CCL) in order to model the breathing of the crack. The CCL is an imaginary line that separates the closed from the open part of the crack. They calculated the variation of the stiffness and flexibility coefficients for differing amount of crack opening and crack depths. Keiner and Gadala [100] presented a two DOF analytical model, a linearized 3D finite element model, and a nonlinear transient 3D FE model to predict the vibration of a cracked shaft under gravity. Modeling and vibration analysis of a simple rotor with a breathing crack is given in [90].
Bachschmid and Tanzi [101] introduced a simplified 1D model of the breathing action of the crack as a function of the static bending moment stresses. They proposed that at the points in the cracked area where the stresses are compressive, contact occurs between the two faces of the crack; where the stresses are instead tensile, no contact occurs. A comparative study (Figure 9) of the shaft displacement, calculated by a 1D model (FLEX), the SERR calculation, and a 3D FEM model, is presented.

*Figure 8 The open-close function of the crack during the rotation of the shaft.*
The integration of the strain energy density function should be performed over the entire area of the cracked section that is under tension, while excluding from the integration the cracked areas under compressive stresses.

The SERR approach does not consider any friction or even impact on the cracked area (during the breathing under loading). These parameters should be taken into consideration in future works. The hysteretic damping of the rotor and the temperature near the crack tip are expected to be influenced by the crack depth, especially under medium and high stresses.

I.2.2.4 Crack Identification Methods

Crack identification in rotating shafts could be realized by applying either vibration- or model-based methods. Vibration methods are based on direct signal measurements such as response or eigenfrequencies; the alteration of these measurements could be assigned to the crack. Model-based methods consider in the place of the crack, or other defects, equivalent loads that give the same result as the crack or the defects do. Sabnavis et al. in [102] published a literature review in 2004 where they concentrated on crack detection in shafts using

(a) Vibration-based methods (signal or model),

(b) Modal testing (eigenmodes or eigenfrequencies changes, response to specially applied excitation) and

(c) Non-traditional methods (neural networks, fuzzy logic, etc.).
**Signal-based Identification**

**Frequency Measurements-based Identification**

When a crack exists in a beam or shaft, then the stiffness is reduced and consequently the eigenfrequencies of the system are decreased. Measuring these differences can help to identify a crack. Nikolakopoulos et al. [103,104] examined the problem of identification of crack depth and position in frame structures, using eigenfrequency measurements. The basic method was to present via contour graph the dependency of the first two structural eigenfrequencies on crack depth and location, using the finite element method. To identify the location and depth of a crack in a frame structure, one only needs to determine the intersecting point of the superposed contours that correspond to the measured eigenfrequency variations caused by the crack presence.

Lee and Chung [105] presented a nondestructive evaluation procedure for identifying a crack in a one-dimensional beam-type structure using the natural frequency data and FEM. Lele and Maiti [106] presented the forward (determination of frequencies of beams knowing the crack parameters) and inverse problem (determination of crack location knowing the natural frequencies) in a Timoshenko beam while representing the crack by a rotational spring. Ratan et al. measured the amplitudes at the peaks of the Fourier transform in order to use them in a “residue” vector to identify and locate the crack [107]. Frequency contours with respect to crack depths and locations can be used to identify the crack [108].

**Eigenmode or Response Based Identification**

Of course the eigenmodes are also modified in cracked vibrating structures. In this case, the vibration measurements at different points could be used for crack identification. In order to identify the location and the depth of a crack in a beam, Rizos et al. [109] used two measurements at two different positions of the beam vibrating near the first resonance; an analytical solution of the dynamic response is also used.

An on-line rotor crack detection and monitoring system have been reported by Imam et al. [110] in 1989. The system is also able to detect cracks during the start-up or shut-down of the machine. This technique is based on the vibration signature analysis approach and on the analytical modeling of the dynamics of the rotor. It is able to detect as small cracks as 1% or 2% of the diameter or the rotor subjected to bending load. Experimental validation of this proposed technique is also included in this circumstantial report. The key crack signatures utilized include both amplitude and phase information of 1/rev, 2/rev and 3/rev harmonics.
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This technology is successfully applied on many turbine-generators systems as well as on nuclear reactor vertical coolant pumps.

Seibold and Weinert [111] presented the localization of cracks in rotating machinery based on measured vibrations. The method used is a time domain identification algorithm: the Extended Kalman Filter (EKF). The localization is performed by designing a bank of EKFs, in which each filter is tuned to a different damage hypothesis: i.e., in this case the specific crack location. By calculating the probabilities of the different hypotheses, the crack can be localized and its depth can be determined. The procedure is applied to a simulated rotor and to a rotor test rig.

Dong et al. [112] used a continuous model for vibration analysis and parameter identification of a static (non-rotating) rotor with an open crack, which is based on two assumptions: that the cracked rotor is an Euler–Bernoulli beam with circular cross section, and that the cracked region is modeled as a local flexibility with fracture mechanics methods. By measuring the deflection at two symmetric points and using the contour method of identification, the crack location and depth are predicted.

Karthikeyan et al. [113] identify the crack in a beam based on free and forced response measurements.

Gounaris and Papadopoulos [114] used a method based on the basic observation that the eigenmodes of any cracked structure are different from those of the uncracked one. The differences are due to the slope discontinuity of the vibration eigenmodes at the crack location. The basic premise in this section is to correlate the mode differences with the crack depth and location. These correlated differences are chosen to be

(a) The ratio of two amplitude measurements in two positions, and

(b) The distance between the node of the vibrating mode and the left end

while the structure is vibrating under harmonic excitation in resonant condition.

Identification Based on External Excitation

A breathing crack existing in a rotating shaft causes nonlinear dynamic behavior. This behavior could be observed, by measuring the vibration amplitude as a function of the angular velocity, during the start-up or shut-down of the rotating machine. Of course it is of great interest the crack identification when the shaft operates at constant angular velocity.

Ishida et al. [115-126] used external excitations of rotating cracked shafts, thus causing the excitation of nonlinear characteristics of the crack in order to identify it. This way the crack can
be detected at the operating rotational speed. The resonances occurring due to the crack at the excitation frequency $\Omega$ correspond to $\pm(\omega \pm \Omega) = p_f, p_b$, $\pm(3\omega \pm \Omega) = p_f, p_b$ and other resonances of the type $\pm(m\omega \pm n\Omega) = p_f, p_b$. The method could be applied in both horizontal and vertical shaft systems.

Instead of FEM, the transfer matrix method can be used to model a cracked Timoshenko shaft. The solution of the problem is given only by (if N=DOF per node) NXN matrix multiplications. In the case of FEM, the dimension of the characteristic matrix is N \times \text{nodes} [127].

The Hwang's and Kim's idea was to minimize the measured and calculated FRF data [128] in order to identify the damage. Simple beam and helicopter rotor blades are used and cracks successfully identified.

Sekhar and Prabhu [129] found the changes of an adequate number of natural frequencies (with FEM) and used these differences to detect the crack. They found that the changes in natural frequencies due to a crack are appreciable in cases of shafts with low slenderness ratio L/D.

In 2001, Saavedra and Cuitino [130,131] calculated the local compliance matrix, then calculated a finite element model of the cracked element, and used it to accomplish detection of cracked beams and rotor.

Vibration peaks occurring at rational fractions of the fundamental rotating critical speed, here named Local Resonances [98], facilitate cracked shaft detection during machine shut-down. A modified Jeffcott-rotor on journal bearings is employed, accounting for gravity effects and oscillating around nontrivial equilibrium points. Modal parameter selection allows this linear model to represent first mode characteristics of real machines. Orbit evolution and vibration patterns are analyzed, yielding useful results. Crack detection results indicate that, instead of 1X$^6$ and 2X components, analysis of the remaining local resonances should have priority; this is due to crack-residual imbalance interaction and to 2x multiple induced origins. Therefore, local resonances and orbital evolution around 1/2, 1/3, and 1/4 of the critical speed are emphasized for various crack-imbalance orientations.

**Coupling-Based Identification**

When a transverse surface crack exists on a shaft, then coupling between different modes of vibration exists. This phenomenon was first reported for longitudinal and bending vibrations.
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by Papadopoulos and Dimarogonas [87] and then for bending and torsional vibrations [88,132].

In [91], the governing field equations for a rotating Timoshenko shaft have been derived, including axial and torsional vibrations as well as boundary conditions. The coupling of bending-torsional and bending-longitudinal is also demonstrated.

The non-diagonal terms of the compliance matrix that models a crack give the ability to numerically express the observations about the existence of coupling between different modes of vibrations. The coupling existence is also proposed as an indication of crack, even if coupling also exists due to geometric or material asymmetries or due to an unbalanced elastic shaft [133]. The coupling due to a crack introduces new discrete characteristics into the vibration spectrum that could be ascribed only to a crack. A crack also alters the mechanical impedance of the vibrating system under an excitation force, thus giving analogous indications [134].

Gounaris et al. [135] used the coupling phenomenon as an identification tool for the determination of the depth and the location of a transverse surface crack in a Timoshenko beam. A harmonic force is used to excite the beam. Two response measurements at a point are required by the method. The first measurement is taken in the direction of the excitation, while the second one in the direction where coupling effect occurs due to the crack. The identification of the existence of the crack will be shown to be feasible if a response exists on a degree of freedom other than the one of the excitation. This method can be applied in structures in air as well as under water.

Gounaris and Papadopoulos [136] used the property of coupled vibrations introduced by a crack in order to identify a crack in a rotating cracked shaft. The shaft was modeled as a rotating Timoshenko beam, including the gyroscopic effect and the axial vibration due to coupling. The method used was based on the measurements of the axial vibration response due to different excitation frequencies and shaft revolutions.

The simple Jeffcott or de Laval rotor model is considered by Darpe et al. in [137] analyzing the response of a rotor with constant angular velocity under axial excitation of different frequencies. This periodic axial impulse is given to both cracked and uncracked rotors and the vibration spectra present clear differences indicating the existence of a crack. Continuing this effort, Darpe et al. [138] examined this phenomenon experimentally. They proved the above analytical results for both rotating and non-rotating shafts, exciting them by axial excitation of different frequencies. Later in 2004, Darpe et al. [139] also investigated the coupling between longitudinal, lateral, and torsional vibrations for a rotating cracked shaft using a response dependent nonlinear breathing crack model. They reported the presence of sums or/and
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differences of frequencies in the vibration spectra that were ascribed to the nonlinearity due to the breathing crack model. They proposed the co-existence of frequencies of other modes in the frequency spectra of a particular mode and the presence of sum and difference frequencies around the excitation frequencies and its harmonics as useful indicators for crack diagnosis.

Dado and Abuzeid [140] presented the vibration behavior of a beam with rectangular cross section carrying end mass and rotary inertia. They investigated the coupling between the longitudinal and bending vibrations and modeled the crack using the 2X2 compliance matrix for axial and bending loads.

Al-Said et al. [141] proposed a simple model that describes the flexural vibration characteristics of a rotating, cracked Timoshenko beam. The cracked beam is modeled using two uniform segments connected by a massless torsional spring at the crack location. The equation of motion is derived using Lagrange’s method in conjunction with the assumed mode method. The proposed model is used to study the effect of crack depth, shear deformation, and rotation speed on the dynamic characteristics of the beam, and to compare the results with those obtained from the widely used Euler–Bernoulli beam model. It is shown that for the same crack depth, the proposed model has a higher reduction in frequency compared to that of the Euler–Bernoulli beam. Model verification is carried out using three-dimensional finite element analysis, which shows good agreement with the assumed mode results.

Model-based Identification

In model-based identification, the fault-induced change of the rotor system is taken into account by equivalent loads in the mathematical model. These equivalent loads are virtual forces and moments acting on the linear undamaged system to generate a dynamic behavior identical to that measured in the damaged system. The method was first presented by Isermann [142] for fault detection of technical processes. This method was applied based on the information extracted from direct measured signals, from signal models, and process models.

Seibold and Weinert [111] presented a localization procedure for cracks in rotating machinery based on measured vibrations. The method used is a time domain identification algorithm: the Extended Kalman Filter (EKF).

Bachschmid and Dellupi [143] used a model based identification procedure to identify the nonlinear forces of linearized and nonlinear oil films in two lobe journal bearings. The method
Presented utilizes the linear model of the rotor and the nonlinear oil-film forces in the identification procedure.

Bachschmid et al. [144] introduced a method based on vibration measurements for the identification of the position and the depth of a transverse crack in a rotor system. A model-based diagnostic approach and a least-squares identification method in the frequency domain are used for the crack localization along the rotor. The crack depth is calculated by comparing the static bending moment, due to the rotor weight and to the bearing alignment conditions, to the identified “equivalent” periodical bending moment which simulates the crack.

The model-based method was also presented by Markert et al. [145] to solve the on-line identification of malfunctions in rotor systems. The fault-induced change in the rotor system is taken into account by equivalent loads. The identification method is based on least squares fitting algorithms in the time domain.

Dharmaraju et al. [146,147] applied inverse engineering techniques to estimate the system model parameters from the experimental force–response measurements. In this work, a general identification algorithm has been developed to estimate crack flexibility coefficients and crack depth based on the force-response information. The transverse surface crack is considered to remain open. The crack has been modeled by a local compliance matrix with four DOF and contains diagonal and off-diagonal terms.

Here, Sekhar [148] applies the model-based ID for a rotor-bearing system, whereas in [149] the rotor bearing system has been modeled using FEM, while the crack is considered through local flexibility change. The crack has been identified as to its depth and location on the shaft. The nature and symptoms of the fault—that is, the crack—are ascertained using the fast Fourier transform. The same author, in 2005 [148], examines the problem of multi-damaged rotor (unbalance and crack) using both modal expansion and reduced basis dynamic expansion.

Pennacchi et al. [150] presented a model-based identification method suitable for industrial machines. The identification method and the relative theory are briefly presented, while three different types of cracks are considered: the first is a slot and therefore not actually a crack since it has not the typical breathing behavior, the second a small crack (14% of the diameter), and the third a deep crack (47% of the diameter). The excellent accuracy obtained in identifying position and depth of different cracks proves the effectiveness and reliability of the proposed method.
**Combined Signal and Model-based Identification**

A combined approach (signal and model-based) for crack identification could improve the accuracy of the results. Feldman and Seibold [151] employed:

(a) signal-based ID: The Hilbert transform was used in order to obtain the signal envelope and the instantaneous frequency, so that various types of nonlinearities due to damage could be identified and classified based on measured response data and

(b) model-based ID: A multi hypothesis bank of Kalman filters is employed for the detection of the size and location of the damage based on the information of the type of damage provided by the results of the Hilbert transform.

A combined (hybrid) model and signal-based approach is also suggested by Sekhar [152] for on-line rotor fault identification. Each fault has been identified for its size and location on the shaft for rotors in transient and steady state situation. The rotor has been modeled using FEM. The nature and symptoms of the fault are ascertained using FFT or Wavelet transforms.

**Other Dynamic Measurements**

It is of great importance that the cracks be identified at the earlier possible stage. Dirr et al. in [153,154] propose an advanced vibration monitoring technique to be used to detect small cracks of at least 5% of the rotor diameter. This technique is based on an angle-dependent sampling of the vibration data and comparison with a reference signal of the undamaged rotor.

**I.2.3 Fluid Film Bearings and Wear**

Hydrodynamic journal bearings are widely used in industry because of their simplicity, efficiency and low cost. They support rotating shafts over a number of years and are often subjected to many stops and starts. During these transient periods, friction is high and the bushes become progressively worn, thus inducing certain disabilities.

Wear in journal bearings is a phenomenon that often occurs in bearings that have been working over long periods (10 years), on huge machinery. Investigation into this phenomenon, which is more prevalent than might have been thought, was first undertaken within the industry because it was a real problem for bearing users.

The onset and development of wear in plain hydrodynamic journal bearings under repeated cycles of starting and stopping has been studied experimentally. The wear which occurred caused easily discernable but localized changes in diametric clearance, surface finish, and
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roundness of the bearing's bore, and these changes were measured after various numbers of operating cycles had been completed. Study of the location, within the bearings of the wear which arose, showed that it was caused entirely by the sliding which occurred during starting and that no significant contribution to the wearing process was made during stopping. It was also observed that, once an initial rapid phase of wearing was completed, the surface finish of the hardened steel shaft reproduced in the regions in the bearing's surface which continued to be worn.

In 1957, Duckworth and Forrester [155] analyzed wear in lubricated bearings. Forrester [156] and Katzenmeier [157] analyzed several examples of damage due to wear quantitatively as well as qualitatively. Another study concerning the problem of wear was led by Dufrane et al. [158] in 1983 who analyzed a worn bearing in a steam turbine. They were the first to propose a geometrical model taking into account the worn region of a bearing, in order to include it in calculations. They paid particular attention to the mechanisms that lead to wear, for a bearing operating at low speed. They showed that wear most often occurs symmetrically on the bottom of the bearing, and that even bearings equipped with an hydrostatic pocket were subjected to wear if the pocket was badly dimensioned.

The first people to be interested in the consequences of a wear defect were Hashimoto et al. [159]. They analyzed the influence of wear defect on the pressure field and on eccentricity ratio and showed that wear defect damages bearing stability and that weak L/D ratio bearing was less sensitive to a defect.

Vaidyanathan and Keith [160] studied the performance of four geometrically different bearings, one of which was a worn bearing. They were interested in the influence of wear defect on parameters such as friction, pressure or the Sommerfeld number compared to the eccentricity ratio. In 1991, Scharrer et al. [161] worked on the dynamic coefficients of a hydrostatic bearing and showed that small wear defects have only a slight influence on bearing performance.

The stability of a worn bearing was then analyzed by Suzuki and Tanaka [162] in 1995. The last work done on the topic is that of Kumar and Mishra [163] who made a study approaching that of Suzuki et al.; they showed that the defect decreases the stability of the hydrostatic bearing when it is submitted to a light load.

In the same year, they continued with their work [164] by analyzing noncircular bearings in turbulent flow submitted to a wear defect in steady-state conditions. They concluded that wear increases the friction as well as the flow rate and reduces the load capacity of the bearing. The few studies carried out on worn bearings are essentially theoretical and do not take into account the thermal effects. Fillon and Bouyer in [165] aimed to analyze the influence of a
wear defect ranging from 10% to 50% of the bearing radial clearance on the characteristics of the bearing such as the temperature, the pressure, the eccentricity ratio, the attitude angle or the minimal thickness of the lubricating film.

In what has with rotor-worn bearing system formulation to do, the works are few. Indicatively, Kumar and Mishra in [163] compound a rigid rotor in turbulent hydrodynamic worn journal bearings and in this analysis a modified Reynolds equation had been used for the unsteady state of the journal. The solution of the dynamic equations had been obtained using the small order perturbation technique and the threshold of oil whirl for a rigid rotor in hydrodynamic worn turbulent journal beatings had been analyzed. The threshold of instability was given by the critical mass parameter and was presented in the form of stability chaffs.

**I.2.4 Fault Features and Multiple Fault Diagnosis in RotorDynamic Systems**

Large rotating machinery such as turbines and compressors are the key equipment in oil refineries, power plants, and chemical engineering plants. Defects and malfunctions (simply called faults) of these machines will result in significant economic loss. Therefore, these machines must be under constant surveillance. When a possible fault is detected, diagnosis is carried out to pinpoint the fault. Diagnosis is very important since different faults require different treatment. For example, surge can be corrected by changing operating parameters while a rotor crack requires an overhaul. Diagnosis is generally much more difficult than detection since different faults may exhibit similar symptoms and several faults may occur at the same time.

Presently, most diagnosis methods are based on pattern matching or pattern classification. First, specific fault patterns are developed, typically based on spectral analysis, and stored in a computer. Then, they are compared to the current pattern. In the past two decades, a large number of pattern matching and pattern classification methods have been developed. Currently, research has been extended to the applications of expert systems, fuzzy systems, and artificial neural networks. However, a successful diagnosis depends greatly on obtaining appropriate fault features and diagnostic indices (the symptoms received from sensor signals). For diagnosis of large rotating machinery, this is particularly important because few models have been developed that can describe the behavior of the machines satisfactorily. In addition, large rotating machinery is usually operated under various disturbances such as temperature, humidity, and time variant and imbalanced load. Hence, sensor signals are often of low signal to noise ratio, which compounds the complexity of the diagnosis. Furthermore, new faults and/or multiple faults emerge from time to time, which generate fault patterns that have never been seen before. Therefore, instead of relying on pattern matching or pattern
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classification, effective fault features and diagnostic indices must be developed. Ideally, diagnostic indices should be:

(a) Sensitive to the faults (ideally with a one-to-one relation)
(b) Insensitive to the variation of machine working conditions
(c) Insensitive to various noise disturbances

For diagnosis of large rotating machinery, according to the survey of recent literature, fault features discovered and diagnostic indices used are primarily based on spectral analysis, which cannot clarify the spatial and temporal information. Phase spectra and holospectra are discussed, but there is no discussion on obtaining quantitative diagnostic indices and diagnostic procedures.

In practice, large rotating machinery may experience various faults such as imbalance, crack, misalignment, rub, surge, oil whirl, rotating stall, fluid excitation, electric power supply fluctuations, loose bearing caps, and pipe excitation. For the diagnosis of these faults, there are a number of frequencies of great importance as listed in Table below.

Note that the surge frequency, oil whirl frequency, rotating stall frequency, loose bearing cap frequency, and pipe excitation frequency are the peak frequencies in specific frequency bands. Their exact values depend on the machine, the machine operating conditions, and the states of the faults. In practice, from a FFT power spectrum, peak frequencies can be found by searching. However, the correlation between a peak frequency and its corresponding fault is usually not readily seen.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Frequency index</th>
<th>Mechanical interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td>1</td>
<td>rotating frequency of the machine</td>
</tr>
<tr>
<td>$f_2$</td>
<td>2</td>
<td>second harmonic of the rotating frequency (2$f_1$)</td>
</tr>
<tr>
<td>$f_3$</td>
<td>3</td>
<td>third harmonic of the rotating frequency (3$f_1$)</td>
</tr>
<tr>
<td>$f_4$</td>
<td>4</td>
<td>fourth harmonic of the rotating frequency (4$f_1$)</td>
</tr>
<tr>
<td>$f_5$</td>
<td>5</td>
<td>fifth harmonic of the rotating frequency (5$f_1$)</td>
</tr>
<tr>
<td>$f_6$</td>
<td>6</td>
<td>sixth harmonic of the rotating frequency (6$f_1$)</td>
</tr>
<tr>
<td>$f_7$</td>
<td>7</td>
<td>surge frequency (peak frequency [0.0-4.0]$f_1$)</td>
</tr>
<tr>
<td>$f_8$</td>
<td>8</td>
<td>oil whirl frequency (peak frequency [0.0-4.0-51]$f_1$)</td>
</tr>
<tr>
<td>$f_9$</td>
<td>9</td>
<td>rotating stall frequency (peak frequency [0.0-7.0-9]$f_1$)</td>
</tr>
<tr>
<td>$f_{10}$</td>
<td>10</td>
<td>loose cap bearing frequency (peak frequency $[0.0-3]$ $f_1$)</td>
</tr>
<tr>
<td>$f_{11}$</td>
<td>11</td>
<td>pipe excitation frequency (peak frequency $[0.0-4.0-5]$ $f_1$)</td>
</tr>
<tr>
<td>$f_{12}$</td>
<td>12</td>
<td>electrical power supply frequency (50 Hz)</td>
</tr>
</tbody>
</table>

Many papers are available in the literature about identification of faults in rotor systems. However, they generally deal only with a single fault, usually an unbalance. Instead, in real machines, the case of multiple faults is quite common: the simultaneous presence of a bow
due to several different causes) and an unbalance or a coupling misalignment occurs often in rotor systems. In [166] an efficient and novel model-based identification method for multiple faults is presented. The method requires the definition of the models of the elements that compose the system, i.e., the rotor, the bearings and the foundation, as well as the models of the faults, which can be represented by harmonic components of equivalent force or moment systems. The identification of multiple faults is made by a least-square fitting approach in the frequency domain, by means of the minimization of a multi-dimensional residual between the vibrations in some measuring planes on the machine and the calculated vibrations due to the acting faults. Some numerical applications are reported for two simultaneous faults and some experimental results obtained on a test-rig are used to validate the identification procedure. The accuracy and limits of the proposed procedure have been evaluated.

In past few years several papers about fault identification have appeared in the literature, dealing with many application fields and introducing different methods. A rather complete survey with a rich bibliography, which ranges in the last 20 years, is reported in [167]. The identification procedure can be performed as usual by means of causality correlations of measurable symptoms to the faults. As regards the rotor dynamics field and limiting to the most recent contributes, two main approaches can be used.

In the first approach, the symptoms can be defined using qualitative information, based on human operators’ experience, which creates a knowledge base. A recent contribution is given in reference [168]: an expert system can be built up in which different diagnostic reasoning strategies can be applied. Fault-symptom matrices, fault-symptom trees, “if-then” rules or fuzzy logic classifications can be used to indicate in a probabilistic approach the type, and sometimes also the size and the location of the most probable fault. Also artificial neural networks (ANN) can be used for creating the symptom-fault correlation. This qualitative diagnostic approach is widely used both in industrial environments and in advanced research work.

The second approach is quantitative and is called the model-based fault detection method. In this case, a reliable model of the system or of the process is used for creating the symptom-fault correlation, or the input-output relation. However, this method has many different ways of application. Among recent contributions available in the literature, Mayes and Penny [169] introduce a fuzzy clustering method in which the basis is to consider the vibration data as a high-dimension feature vector and the vibration caused by a particular fault on a specific machine can considered being a point in this high-dimension space. This same fault, on a number of similar machines, should produce a cluster of points in the high-dimension space that is distinct from other clusters produced by different faults. The main drawback of this
method is the availability of a large database of the dynamic behavior of similar machines, which can emphasize the differences in the response of similar machines.

In other applications, the fault detection can be performed by means of different model-based approaches, according to the nature of the system under observation:

Parameter estimation, when the characteristic constant parameters of the process or of the components are affected by the fault.

State estimation, when the constant parameters are unaffected by possible faults and only the state of the system, which is represented by a set of generally immeasurable state variables (function of time), is affected by the faults; in this case the model acts as a state observer.

Parity equations, when the faults affect some of the immeasurable input variables, the parameters are constant and only output variables are measured and compared with calculated model output variables.

Therefore, the fault can be identified from parameter or state estimation or from parity equations.

Kreuzinger-Janik and Irretier [170] use a modal expansion of the frequency response function of the system, on both numerical model and experimental results, to identify the unbalance distribution of a rotor. Markert et al. [171] and Platz et al. [172] present a model in which equivalent loads due to the faults (rubbing and unbalances) are virtual forces and moments acting on the linear undamaged system model to generate a dynamic behavior are identical to the measured one of the damaged system. The identification is then performed by least-squares fitting in the time domain. Edwards et al. in [173] employ a model-based identification in the frequency domain to identify an unbalance on a test-rig.

A more comprehensive approach, able to identify several different types of faults and to discriminate among faults which generate similar harmonic components, is been introduced by Bachschmid and Pennacchi [174]. This method has been experimentally validated on different test-rigs and real machines (see also references [175-178]) for many types of faults, such as unbalances, rotor permanent bows, rotor rubs, coupling misalignments, cracks, journal ovalization and rotor stiffness asymmetries. In this model-based identification procedure, the input variables are the exciting forces and the output variables are the vibrations. The procedure requires the model definition of the elements (rotors, bearings, and supporting structure) that compose the rotor system. A finite beam element model is assumed for the rotor, the bearings are represented by means of their stiffness and damping matrices (therefore, non-linear oil film effects are neglected), while several representations can be given for the foundation, such as modal, elasto-dynamic matrix or lumped springs and
dampers. Also, the effect of the faults has to be modeled and this is done by introducing an equivalent system of external forces and moments. In [179], the method is improved in order to identify simultaneously two or more faults acting on a rotor, since the case of multiple faults may occur in real machines: sometimes a bow (due to several different causes) and an unbalance or a coupling misalignment may develop simultaneously.

Generally, the fault identification procedure is started when the vibration vector change exceeds a suitable pre-established acceptance region; in this case, it is more likely that the change in the vibration behavior is really caused by an impeding fault only. But when the reference situation is not available, the arising fault is superposed to the original unbalance and bow distribution. In this case also, the multiple fault identification may be useful for selecting the simultaneous faults.
I.3. The Current Study

I.3.1 Assumptions and Methodology

In this work there is an amount of assumptions other based on previous works and some others to provide different approximations developed during the current study. During the next chapters, the detailed definition of recent approximations will be provided. Also in the next paragraph section (I.3.2) the points of novelty will be defined in brief. For the time, in this section, the basic assumptions, as long as, the methodology of current study is mentioned in brief in the main directions of: the shaft model, the crack model, the bearing model, the nonlinearity of the entire system, the solution and the proposed detection method. In continue, the way that each of the above directions was confronted is given by describing in brief each chapter.

Chapter 1

This chapter is the point of the current work that incorporates the rotor bearing system in its full form. In brief, the fluid film bearings have definable geometric and physical properties mounting the rotor with nonlinear fluid film forces that become a function of systems response\(^7\). The bearings are assumed as “Finite bearings” avoiding the assumption of “Long” or “Short”\(^8\) bearings, towards generality, and finite difference method is used for the solution of Reynolds equation of fluid film pressure for laminar, isothermal and isoviscous flow of the lubricant.

The fluid film forces are evaluated at every time step during response and applied to the rotors ends with specified boundary conditions. In order to achieve results during critical speed the internal hysteretic damping\(^9\) of the shaft is incorporated in the analysis in a way of Real number modification\(^10\). With the boundary conditions of shearing force to become functions of systems response in the journal points the system becomes dynamic resulting sometimes in no synchronous response even with the excitation of constant drive frequency. The solution of the system is achieved with a modified Newton Raphson method\(^11\). Rotor bearing systems with properties taken from literature are used so as to present the progress of current algorithm in with some time series about transient and steady state response to be presented. Using the current results some remarks are made about the quality of extracted time series and the components that are enclosed in response in various speed operation, so as to present the bearing nonlinearity.
Introduction

In this initial part of the entire work no defects are introduced in the system and the results are of intact system simulation, but since the subject of bearing is externally remarked in this chapter the confrontation of worn bearing is presented. In this simulation the defects of crack and wear will be introduced in chapters 5 and 6 correspondingly.

Chapter 2

This chapter inspects in detail the local compliance that the transverse crack introduces in the system. The local compliance matrix includes four magnitudes (compliances) that express two direct compliances, one for vertical and one for horizontal plane, and two cross coupled compliances, one for coupling from horizontal plane to vertical plane, and the other one for coupling from vertical plane to horizontal plane. Further definition is made in the current chapter. The local compliance matrix is of dimension 2X2 and is defined as a function of crack rotational angle and crack depth.

The local compliances are calculated for various rotational angles for which the form of the crack offers the use of Stress Intensity Factors (SIF). This assumption implies that there are angles of rotation in which the compliances are calculated by interpolating the already known values (those calculated using SIF). The form of the crack is defined with the assumption that the static bending moment produced from the rotors weight is greater than any other bending moment produced from the rest loads during operation and thus the total bending moment direction remains steady producing alternative bending in the cracked section during rotation.

The current assumption for the crack breathing definition introduces periodic local stiffness coefficients in the system and thus the system is characterized as linear system with periodically variable coefficients. Further definition in order to finally achieve a more realistic model of the crack behavior is made in this chapter by interacting the rotor response and rotor loading with the developed bending moments of the cracked section. This assumption, which finally is used in this dissertation, expresses the bending moments produced by rotors weight, unbalance force and rotors response as a function of crack form and thus the crack can obtain steady or variant form during operation regarding the resonance phenomena and the loading magnitudes also.

The main result of this chapter is the definition of crack local compliance matrix as a function of crack depth and rotational angle or in other words the “breathing” mechanism of the current crack model.
Chapter 3

A continuous, static, clamped-free, Euler-Bernoulli Beam of circular section and submitted to bending (exclusively) load is considered as the model in which the crack defect will be introduced in order to observe the compliance variation effects during discrete rotation. Here, the domination of the gravity into the vibration response of the system is assumed. This assumption provides that, no matter the response amplitude is, the crack form does not change. To explain further, the crack tips are coming closer and farer during vibration but do not touch each other. So the crack form is definitely determined from the parameter of static load due to weight. This assumption does correspond to real vibratory cases of large and heavy rotors of high L/D ratio usually being met in real turbomachines.

Since the crack form and in extent, the local compliance, and the total stiffness of the system remain constant during vibration, the system is linear and a gauss elimination method is used to obtain the response. Many other results such as eigenfrequencies, response amplitude, eigenmodes etc are obtained during the cracks rotation. The vibration analysis is performed initially only in one plane of vibration, and this is the vertical, since the gravity effect is included.

In this chapter the concept of rotational speed is concerned also with a continuous rotor to spin and whirl in the same speed under the assumption of Rayleigh system of equations of motion for both main planes, vertical and horizontal. Transverse shear effect, gyroscopic effects and axial torque are considered at this coupled system of equations.

The defect of crack is introduced this time in a rotating system and the result is a time-varying local compliance and total stiffness of the shaft. The problem is considered as non-linear due to the variation of system stiffness properties during rotation. The fundamental solution of the Rayleigh system equations is solved in discrete time with the crack - boundary conditions to become time-depended due to the crack rotation. The Newton-Raphson method is used to achieve the definition of constant parameters of general solution.

One of the targets in this chapter is to obtain results for the effect of breathing crack coupling in synchronous whirling response under the excitation of unbalance and of full stiffness bearing assumption. Steady state and transient response during start up is obtained and frequency domain measurements are discussed in relationship to the crack coupling effect.

In continue the bearings are introduced with the simple assumption of linear stiffness and damping coefficients in order to observe the coupling response characteristics due to the three simultaneous reasons of coupling: equations of motion, crack and bearings. Vibration
amplitude is calculated as a function of rotational speed for various crack depths and the crack coupling is observed among the other two coupling effects in the response. Also, time response-rotor orbits are calculated and compared for the cases of the intact and the crack system among other results.

Chapter 4

A detailed presentation of the experimental layout that was constructed during this dissertation is made in this chapter. The experimental rotor bearing system was constructed in order to fulfill the demands of comparison of experimental and numerical time series for the cases of intact and defected system. The experimental results for the cracked and worn system are presented in chapters 5 and 6 correspondingly together with the respective numerical results. The experimental set up was used for the vertical and horizontal response acquisition in the points of bearing and disc in order to inspect:

1) The effects of crack in the development of non synchronous with the rotation harmonic components in transient and steady state.

2) The effects of wear in the development of non synchronous with the rotation harmonic components in transient and steady state in order to match numerical results that can be considered for wear detection.

3) The extraction and isolation of signals provoked from the coupling due to crack so as to validate the method for crack detection using external excitation.

4) The periodic and quasi-periodic oscillations during normal operation of the system with constant rotational speed and the effects of crack and wear in the periodicity of the whirling.

Chapter 5

This chapter is dedicated to the effects that the crack presence reveal under several concept analysis such as various speed amplitude measurements, steady state and transient time histories, Poincaré maps, fractal dimension, coherency measurements, Time - Frequency analysis, critical speeds and other magnitudes that can be extracted in order to obtain the observations and the mechanism that provides them according to the defect of crack.

The results are confirmed using a real experimental rotor bearing system that was set up individually during this work. There is an externally detailed reference about the design, and manufacturing of the rotor bearing system, the lubricating system and the data acquisition concept of the machine in Appendix C. The results obtained in chapters 5, 6 and 7 matches
those from the experimental analysis providing the reliability of the current simulation as long as of current detection method presented in chapter 7.

Chapter 6

In this chapter, significant observations about the bearing wear effects in rotor bearing system transient and steady state response are presented under the concept of bearing wear detection. A similar to chapter 5 analysis has to do again with the concepts of various speed amplitude measurements, steady state and transient time histories, Poincaré maps, fractal dimension, coherency measurements, Time - Frequency analysis and critical speeds. The results are compared again with the corresponding results of the experimental system that for the current scope included a worn fluid film bearing. The detailed experimental progress for the case of worn bearing is presented in this chapter as long as in Appendix C also.

Chapter 7

In chapter 7 a method for crack and wear detection is presented. The concept of the method is to provoke the symptoms that the crack and the wear produces when an external excitation force is applied under the normal steady state operation of the machine without to “disturb” the normal operation of the system.

The main aim is to detect a crack by isolating and extracting the coupling due to it. Also, the system operation is assumed to be of steady state and the external excitation is assumed to be harmonic and is applied electromagnetically in the shaft. The electromagnetic excitation is of constant direction, in horizontal plane, applying a harmonic force of either linear frequency variation or constant frequency. The external excitation provides the amplification of nonlinear resonances due to crack [122] and is used to provoke the crack coupling of vertical and horizontal direction. The method is presented analytically in chapter 7 together with various measures of steady state and transient response of the cracked and worn system, and is judged efficient to identify cracks of depth even of 10% of shaft radius and wear of depth 20% of bearing radial clearance.

The rotor bearing system simulation of chapter 4 is used since it concerns the entire coupling and other phenomena incorporation and thus the results are in a good agreement with the corresponding produced from the experimental procedure.
The general progress of current work

Through this above brief presentation of current work, a conclusion about the methodology is that the problem was seriatim build up from specific to more general concepts:

- the crack compliance → the crack coupling → the rotating shaft coupling due to crack → the linear bearings rotor system with coupling due to several parameters → the nonlinear finite bearing rotor system with crack and wear → the observations of each defect effects → the use of external excitation in the nonlinear rotor bearing system → the fault detection concept.

The entire procedure and methodology (see also Figure 10) was planned during the work, with the aspect of coupling to become at last a significant matter in crack detection. Also, the last results of worn system gave the ability to develop criteria for wear defect detection so as the concept of multiple faults to be included in this dissertation. In following section, the contribution of this work is presented in brief.

![Figure 10. Dissertation flow chart.](chart.png)

I.3.2 Contribution

The novelty and contribution of this dissertation in the entire field of rotor dynamics with defects can be briefly mentioned as:
The coupling compliance estimation for a breathing crack model using SIF method for an entire rotation of the crack. **Paper 1**, (see Short Biography).

The modeling of coupled bending vibrations of a stationary shaft under the current assumption of coupling compliance. **Paper 2 & 3**, (see Short Biography).

The modeling of a continuous Rayleigh cracked rotor in linear bearings with coupling coefficients. **Paper 4 & 7**, (see Short Biography).

The development of a nonlinear dynamic rotor- finite bearing system with the effects of breathing crack. **Paper 5**, (see Short Biography).

The combination of a Rayleigh shaft with worn finite fluid film bearings and the effects of wear in nonlinear response. **Paper 6 & 8**, (see Short Biography).

The coupling based method using external excitation for crack detection in nonlinear rotor bearing systems. **Paper 9**, (see Short Biography).

The use of the effects of bearing wear in nonlinear rotor bearing systems as a diagnostic tool for the bearing wear detection using external excitation. **Paper 10**, (see Short Biography).
Chapter 1

A Nonlinear, Dynamic, Continuous, Damped Model for the Simulation of Rotor Bearing-System Oscillations

A rotor bearing system consisting of a continuous rotor and finite fluid film bearings is developed in this chapter. The current rotor bearing system construction is achieved using boundary conditions that combine the rotor’s shearing force and the fluid film forces at the points where the bearings are located. The bearing behavior is considered to be nonlinear since the dynamic properties of the bearing are an actual function of journal position and journal linear velocity. The nonlinear consideration in the journal bearings gives the analysis a highly improved level of precision, much higher than linear analysis is able to give, especially for the case of rotational speed near the critical speed, which provokes journal displacements near the radial clearance. For this reason, material damping is introduced in the rotor model so as to achieve solutions in the time domain, even at critical speeds, which are independent from the amount of time that the system exists at resonance. In such cases, linear bearing consideration gives results of response very close to or even higher than radial clearance and is judged as insufficient. The mathematical model expresses an initial value and boundary condition problem with time-dependent boundary conditions that are also functions of initial values; this is a result of expressing the fluid film forces as a function of the rotor’s shearing force. The main target of the current work is to investigate the system’s dynamic characteristics in both frequency and time domains under the assumption of continuous rotor on nonlinear bearings.
1.1 Formulation of shaft motion

1.1.1 Equations of motion

In this approach to the rotor motion, the rotary inertia, the shear deformation, the torque due to power transmission, and the gyroscopic effect are considered. Suppose a homogeneous multi-step rotor consists of 2 steps, as shown in Figure 1.1, with a complex Young modulus \( E' = E(1+i\cdot \eta) \), complex shear modulus \( G' = G(1+i\cdot \eta) \), material loss factor \( \eta \), polar moment of inertia of each cross section \( I_j \) (\( j = 1, 2 \)), mass density \( \rho \), form factor \( k = 10/9 \) for circular cross section, length of each step \( L_j \), radius of each step \( R_j \), cross section area of each step \( A_j \), radius of gyration of each step \( r_{0j} = \sqrt{I_j/A_j} \), and Poisson ratio \( v \). It rotates with a rotational speed \( \Omega \), whirls with a whirling frequency \( \omega \), and transmits a power with an axial torque \( T \). If \( Y_j(x,t) \) and \( Z_j(x,t) \) are the vertical and horizontal responses, with respect to the coordinate system \( Oxyz \) defined in Figure 1.1, with the axial coordinate \( x \) and time \( t \), respectively, then the coupled governing equation of motion of each step can be written as in [22,23,176] in Equations (1.1) and (1.2) for the vertical and horizontal plane, respectively.

\[
E' \frac{\partial^4 Y_j}{\partial x^4} - \frac{E' I_j}{k G' \frac{\partial^3 Y_j}{\partial t^3}} + \frac{T \rho}{k G' \frac{\partial^3 Z_j}{\partial t^3}} + \frac{\rho A_j}{k G' \frac{\partial^3 Y_j}{\partial t^3}} - \rho A_j r_{0j} \left[ \frac{\partial Y_j}{\partial x} - \frac{\partial Y_j}{\partial x} \right] + 2\Omega \left( \frac{\partial^2 Y_j}{\partial x \partial t} - \frac{\partial Y_j}{\partial x} \right) = 0
\]

\[
E' \frac{\partial^4 Z_j}{\partial x^4} - \frac{E' I_j}{k G' \frac{\partial^3 Z_j}{\partial t^3}} - \frac{T \rho}{k G' \frac{\partial^3 Y_j}{\partial t^3}} + \frac{\rho A_j}{k G' \frac{\partial^3 Z_j}{\partial t^3}} - \rho A_j r_{0j} \left[ \frac{\partial Z_j}{\partial x} - \frac{\partial Z_j}{\partial x} \right] - 2\Omega \left( \frac{\partial^2 Z_j}{\partial x \partial t} - \frac{\partial Z_j}{\partial x} \right) = 0
\]

Equations (1.1) and (1.2) consist of a system of fourth-order partial differential equations which have solutions with a real and imaginary part. Assuming a solution of the form \( Y_j(x,t) = y_j \cdot e^{i \omega t} \cdot e^{i \omega t} \) and \( Z_j(x,t) = z_j \cdot e^{i \omega t} \cdot e^{i \omega t} \), and substituting in the differential Equations
(1.1) and (1.2), two algebraic equations of magnitudes \( y_j \) and \( z_j \) are formed, as in Equations (1.3) and (1.4), resulting in a new algebraic system of unknowns \( y_j \) and \( z_j \).

\[
\beta_1 \lambda^4 + \beta_2 \lambda^3 + \beta_3 \lambda^2 + \beta_4 \lambda + \beta_5 = 0 \tag{1.3}
\]

\[
\beta_6 \lambda^4 + \beta_7 \lambda^3 + \beta_8 \lambda^2 + \beta_9 \lambda + \beta_{10} = 0 \tag{1.4}
\]

Figure 1.1 a) Rotor with a disk mounted on two fluid film bearings. b) The current consideration for the simulation of a rotor bearing system.
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Variables $\beta_i, i=1,2,...,10$ are defined in Equations (1.5)-(1.14).

\[
\beta_i = E I_j y_j (1 + i \eta) \quad (1.5)
\]

\[
\beta_2 = T z_j \quad (1.6)
\]

\[
\beta_3 = \rho \Omega^2 \left( A_j k r_{0j}^2 + 2 I_j (1 + \nu) \right) y_j - 2 i A_j k r_{0j}^2 z_j / k \quad (1.7)
\]

\[
\beta_4 = - \frac{2 i T (1 + \nu) \rho \Omega^2 z_j}{E k (\eta - i)} \quad (1.8)
\]

\[
\beta_5 = A_j \rho \Omega^2 \left( \left( E k (\eta - i) + 2 i r_{0j}^2 (1 + \nu) \rho \Omega^2 \right) y_j + 4 r_{0j}^2 (1 + \nu) \rho \Omega^2 z_j \right) / E k (\eta - i) \quad (1.9)
\]

\[
\beta_6 = E I_j z_j (1 + i \eta) \quad (1.10)
\]

\[
\beta_7 = - T y_j \quad (1.11)
\]

\[
\beta_8 = \rho \Omega^2 \left( A_j k r_{0j}^2 + 2 I_j (1 + \nu) \right) z_j + 2 i A_j k r_{0j}^2 y_j / k \quad (1.12)
\]

\[
\beta_9 = \frac{2 i T (1 + \nu) \rho \Omega^2 y_j}{E k (\eta - i)} \quad (1.13)
\]

\[
\beta_{10} = \frac{A_j \rho \Omega^2 \left( \left( E k (\eta - i) + 2 i r_{0j}^2 (1 + \nu) \rho \Omega^2 \right) z_j + 4 r_{0j}^2 (1 + \nu) \rho \Omega^2 y_j \right)}{E k (\eta - i)} \quad (1.14)
\]

In order to achieve a non-zero solution of the system in Equations (1.3) and (1.4), there must be a zero characteristic determinant. The determinant of the characteristic matrix $\Delta$ of the system in Equations (1.3) and (1.4) is composed as in Equation (1.15).

\[
\Delta = \begin{pmatrix}
a_{1,1} & a_{1,2} \\
a_{2,1} & a_{2,2}
\end{pmatrix} \quad (1.15)
\]
Variables \( a_{ij}, i, j = 1, 2 \) are defined in Equations (1.16)-(1.18).

\[
a_{1,1} = a_{2,2} = \frac{i E^2 I_j k (\eta - i)^2 \lambda^4 - 2 i A_j r_0^2 (1 + v) \rho^2 \Omega^4 + E (\eta - i) (2 I_j (1 + v) \lambda^2 + A_j k \lambda^2 \lambda^2 - 1) \rho \Omega^2}{E k (\eta - i)}
\]

(1.16)

\[
a_{1,2} = \frac{\left(T \lambda - 2 i A_j r_0^2 \rho \Omega^2\right)\left(E k (\eta - i) \lambda^2 - 2 i (1 + v) \rho \Omega^2\right)}{E k (\eta - i)}
\]

(1.17)

\[
a_{2,1} = -\frac{\left(T \lambda - 2 i A_j r_0^2 \rho \Omega^2\right)\left(E k (\eta - i) \lambda^2 - 2 i (1 + v) \rho \Omega^2\right)}{E k (\eta - i)}
\]

(1.18)

The eighth-order characteristic polynomial \( \text{Det}(\Lambda) = a_{1,1} a_{2,2} - a_{1,2} a_{2,1} \) becomes as in Equation (1.19).

\[
\gamma_1 \lambda^8 + \gamma_2 \lambda^7 + \gamma_3 \lambda^6 + \gamma_4 \lambda^5 + \gamma_5 \lambda^4 + \gamma_6 \lambda^3 + \gamma_7 \lambda^2 + \gamma_8 \lambda + \gamma_9 = 0
\]

(1.19)

Variables \( \gamma_i, i = 1, 2, ..., 8 \) are defined in Equations (1.20) - (1.27).

\[
\gamma_1 = -E^2 I_j^2 y_j (\eta - i)^2
\]

(1.20)

\[
\gamma_2 = \left(4 E I_j^2 (1 + v)(1 + i \eta) \rho \Omega^2 + k \left(T^2 + 2 A_j E I_j r_0^2 (1 + i \eta) \rho \Omega^2\right)\right) / k
\]

(1.21)

\[
\gamma_3 = -4 i A_j r_0^2 T \rho \Omega^2
\]

(1.22)

\[
\gamma_4 = \frac{\rho \Omega^2 A_j E k}{E k^2 (\eta - i)} \left(3 A_j k r_0^2 (\eta - i) \rho \Omega^2 + 2 I_j (\eta - i)\left(E k (1 + i \eta) - 4 r_0^2 (1 + v) \rho \Omega^2\right)\right)
\]

(1.23)

\[
\gamma_5 = -\frac{16 A_j r_0^2 T (1 + v) \rho^2 \Omega^4}{E k (\eta - i)}
\]

(1.24)
\[
\gamma_6 = \frac{2 \rho^2 \Omega^4}{E^2 k^2 (\eta-i)^2} \left( 2 T^2 (1+v)^2 + A_j E k r_0^2 (\eta-i) \left( E k (\eta-i) - 6 i r_0^2 (1+v) \rho \Omega^2 \right) \right) \\
+ \frac{4 \rho^2 \Omega^4 A_j E I}{E^2 k^2 (\eta-i)^2} (1+v)(1+i\eta) \left( -E k (1+i\eta) + 2 r_0^2 (1+v) \rho \Omega^2 \right) \\
\] (1.25)

\[
\gamma_5 = \frac{16 i A_j r_0^2 T (1+v)^2 \rho^4 \Omega^6}{E^2 k^2 (\eta-i)^2} \\
\] (1.26)

\[
\gamma_8 = \frac{A_j \rho^2 \Omega^4}{E^2 k^2 (\eta-i)^2} \left( E^2 k^2 (\eta-i)^2 + 4 E k r_0^2 (1+v)(1+i\eta) \rho \Omega^2 + 12 r_0^2 (1+v)^2 \rho^2 \Omega^4 \right) \\
\] (1.27)

Neglecting the torque \( T \) due to power transmission (\( T = 0 \)) the terms \( \gamma_3, \gamma_5, \gamma_7 \) become zero and the characteristic polynomial in Equation (1.19) is solved yielding the eight eigenvalues as shown in Equations (1.28) - (1.31).

\[
\lambda_{1,2} = \mp \sqrt{\delta_1 - \frac{1}{2} (\sqrt{\delta_1^2} + \sqrt{\delta_1^2 - \delta_1^4})} \\
\] (1.28)

\[
\lambda_{3,4} = \mp \sqrt{\delta_1 - \frac{1}{2} (\sqrt{\delta_1^2} - \sqrt{\delta_1^2 - \delta_1^4})} \\
\] (1.29)

\[
\lambda_{5,6} = \mp \sqrt{\delta_1 + \frac{1}{2} (\sqrt{\delta_1^2} - \sqrt{\delta_1^2 + \delta_1^4})} \\
\] (1.30)

\[
\lambda_{7,8} = \mp \sqrt{\delta_1 + \frac{1}{2} (\sqrt{\delta_1^2} + \sqrt{\delta_1^2 + \delta_1^4})} \\
\] (1.31)

Variables \( \delta_i \), \( i = 1, 2, \ldots, 6 \) are defined in Equations (32)-(37).

\[
\delta_1 = -\gamma_2 / (4 \gamma_1) \\
\] (1.32)

\[
\delta_2 = \frac{\gamma_2^2}{4 \gamma_1^2} - \frac{2 \gamma_2}{3 \gamma_1} + \frac{2^{\frac{1}{3}} (\gamma_2^2 - 3 \gamma_3 \gamma_6 + 12 \gamma_1 \gamma_8)}{3 \gamma_1 \delta_3^{\frac{1}{3}}} + \frac{\delta_3^{\frac{1}{3}}}{3 \gamma_1 2^{\frac{1}{3}}} \\
\] (1.33)
\[
\delta_3 = \frac{\gamma_2^2}{2\gamma_1^2} - \frac{2\gamma_1}{3\gamma_1} \left( \frac{\gamma_2^2 - 3\gamma_2 \gamma_6 + 12\gamma_1 \gamma_6}{3\gamma_1 \delta_3^{1/3}} \right) - \frac{\delta_3^{1/3}}{3\gamma_1^{1/3}} \tag{1.34}
\]

\[
\delta_4 = -\frac{\gamma_1^3 + 4\gamma_2 \gamma_4 - 8\gamma_6}{\gamma_1^3} \tag{1.35}
\]

\[
\delta_5 = \delta_6 + \sqrt{-4\left( \gamma_1^2 - 3\gamma_2 \gamma_6 + 12\gamma_1 \gamma_6 \right)^3 + \delta_6^2} \tag{1.36}
\]

\[
\delta_6 = 2\gamma_4^2 - 9\gamma_2 \gamma_4 \gamma_6 + 27\left( \gamma_1 \gamma_6^2 + \gamma_4 \gamma_2^2 \right) - 72\gamma_4 \gamma_6 \tag{1.37}
\]

The characteristic roots \( \lambda_i, i = 1, 2, ..., 8 \) are complex numbers, and in the case where the torque \( T \) is set equal to zero, the roots are conjugate complex numbers in pairs of two. Thus, the solutions of the differential equations in Equations (1.1) and (1.2) become as in Equations (1.38) - (1.41) for the first step, and as in Equations (1.42) - (1.45) for the second step of the rotor. The solutions are real numbers and express the real and the imaginary parts of the responses. This formulation in real and imaginary parts is needed so as to achieve expressions of the responses at the location of the bearings, as will be shown in the section on boundary conditions. The need for expressing the response in real numbers cannot be avoided, and introduces a complexity, and as a result, an algorithmic cost in time evaluation. However, there is not a way to introduce bearing forces in complex form since the bearing forces are obtained using the finite differences method, without using stiffness and damping coefficients, as has been widely applied in other simulations in literature.

\[
Y_{x,1}(x,t) = \text{Re}[Y_1(x,t)] = e^{-\lambda_{1,x}t} \left( q_1 \cos \lambda_{1,x}x + q_6 \sin \lambda_{1,x}x \right) + e^{\lambda_{1,x}t} \left( q_4 \cos \lambda_{1,x}x + q_8 \sin \lambda_{1,x}x \right) + e^{i\lambda_{1,x}t} \left( q_2 \cos \lambda_{1,x}x + q_{10} \sin \lambda_{1,x}x \right) \tag{1.38}
\]

\[
Y_{x,2}(x,t) = \text{Im}[Y_1(x,t)] = e^{-\lambda_{1,x}t} \left( q_1 \sin \lambda_{1,x}x + q_6 \cos \lambda_{1,x}x \right) + e^{\lambda_{1,x}t} \left( q_4 \sin \lambda_{1,x}x + q_8 \cos \lambda_{1,x}x \right) + e^{i\lambda_{1,x}t} \left( q_2 \sin \lambda_{1,x}x + q_{10} \cos \lambda_{1,x}x \right) \tag{1.39}
\]

\[
Z_{x,1}(x,t) = \text{Re}[Z_1(x,t)] = e^{-\lambda_{1,x}t} \left( q_1 \cos \lambda_{1,x}x + q_7 \sin \lambda_{1,x}x \right) + e^{\lambda_{1,x}t} \left( q_4 \cos \lambda_{1,x}x + q_{11} \sin \lambda_{1,x}x \right) + e^{i\lambda_{1,x}t} \left( q_2 \cos \lambda_{1,x}x + q_{13} \sin \lambda_{1,x}x \right) \tag{1.40}
\]

\[
Z_{x,2}(x,t) = \text{Im}[Z_1(x,t)] = e^{-\lambda_{1,x}t} \left( q_1 \sin \lambda_{1,x}x + q_7 \cos \lambda_{1,x}x \right) + e^{\lambda_{1,x}t} \left( q_4 \sin \lambda_{1,x}x + q_{11} \cos \lambda_{1,x}x \right) + e^{i\lambda_{1,x}t} \left( q_2 \sin \lambda_{1,x}x + q_{13} \cos \lambda_{1,x}x \right) \tag{1.41}
\]
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\[ Y_{2,i} (x,t) = \text{Re}\{Y_2 (x,t)\} = e^{-\lambda_i x} \left( q_{i1} \cos \lambda_i x - q_{i2} \sin \lambda_i x \right) + e^{\lambda_i x} \left( q_{i3} \cos \lambda_i x + q_{i4} \sin \lambda_i x \right) \]

\[ + e^{-\lambda_i x} \left( -q_{i5} \sin \lambda_i x + q_{i6} \cos \lambda_i x \right) + e^{\lambda_i x} \left( q_{i7} \sin \lambda_i x + q_{i8} \cos \lambda_i x \right) \]

\[ Y_{2,i} (x,t) = \text{Im}\{Y_2 (x,t)\} = e^{-\lambda_i x} \left( q_{i9} \sin \lambda_i x - q_{i10} \cos \lambda_i x \right) + e^{\lambda_i x} \left( -q_{i11} \sin \lambda_i x + q_{i12} \cos \lambda_i x \right) \]

\[ + e^{-\lambda_i x} \left( q_{i13} \cos \lambda_i x + q_{i14} \sin \lambda_i x \right) + e^{\lambda_i x} \left( -q_{i15} \cos \lambda_i x + q_{i16} \sin \lambda_i x \right) \]

\[ Z_{2,i} (x,t) = \text{Re}\{Z_2 (x,t)\} = e^{-\lambda_i x} \left( q_{i17} \cos \lambda_i x - q_{i18} \sin \lambda_i x \right) + e^{\lambda_i x} \left( q_{i19} \cos \lambda_i x + q_{i20} \sin \lambda_i x \right) \]

\[ + e^{-\lambda_i x} \left( -q_{i21} \sin \lambda_i x + q_{i22} \cos \lambda_i x \right) + e^{\lambda_i x} \left( q_{i23} \sin \lambda_i x + q_{i24} \cos \lambda_i x \right) \]

\[ Z_{2,i} (x,t) = \text{Im}\{Z_2 (x,t)\} = e^{-\lambda_i x} \left( q_{i25} \sin \lambda_i x - q_{i26} \cos \lambda_i x \right) + e^{\lambda_i x} \left( -q_{i27} \sin \lambda_i x + q_{i28} \cos \lambda_i x \right) \]

\[ + e^{-\lambda_i x} \left( q_{i29} \cos \lambda_i x + q_{i30} \sin \lambda_i x \right) + e^{\lambda_i x} \left( -q_{i31} \cos \lambda_i x + q_{i32} \sin \lambda_i x \right) \]

Variable \( \lambda_{ij}, i = 1,2,3,4, j = R,I \) are defined in Equations (1.46) and (1.47); note that they are equal for both steps unless the physical and geometric properties of each step are different.

\[ \lambda_{iR} = -\text{Re}\{\lambda_i\}, \quad \lambda_{iI} = \text{Im}\{\lambda_i\}, \quad \lambda_{2,iR} = -\text{Re}\{\lambda_2\}, \quad \lambda_{2,iI} = \text{Im}\{\lambda_2\} \]

\[ \lambda_{3,iR} = -\text{Re}\{\lambda_3\}, \quad \lambda_{3,iI} = \text{Im}\{\lambda_3\}, \quad \lambda_{4,iR} = -\text{Re}\{\lambda_4\}, \quad \lambda_{4,iI} = \text{Im}\{\lambda_4\} \]

The supposed constants \( q_i, i = 1,2,\ldots,32 \) are functions of time \( q_i(t) \), but they can be defined as constants for every time step of the evaluation.

The real and imaginary parts of the bending moment and shearing force are given for each step of the rotor for both planes. The current analysis uses a different expression for the equation of motion excluding the time \( t \). The partial solution of time in Equations (1.3) and (1.4) is known as \( T(t) = B_1 \cos(\omega t) + B_2 \sin(\omega t) \) for a harmonic motion. This solution demands boundary conditions invariable with \( x \) only so as to define constants \( q_i \), and initial conditions of variable \( t \) so as to define constants \( B_1 \) and \( B_2 \). This formulation yields a harmonic response that cannot express the current solution. From the current point of view, the boundary conditions are functions of variables \( x \) and \( t \), so the differential equations in Equations (1.1) and (1.2) are unsolvable under the assumption that the partial time solution is a harmonic expression. The response of systems with dynamic boundary conditions is an addition of the steady-state response and the transient response, but there exist physical problems that
cannot be guided to steady-state solutions. For example, a startup of a rotor bearing system always has a transient response.

In the current procedure, there is the assumption of excluding the partial time solution, and the general solution is expressed by replacing the constants $q_i$ of unknown functions of time $q_i(t)$. There is no need for expressions $q_i(t)$ to be known, because for a specific value of time $t_i$, the functions $q_i(t)$ can obtain constant values that can be calculated from the boundary conditions.

1.1.2 Boundary conditions

The boundary conditions express the continuity and the discontinuity of shearing force, bending moment, slope and displacement of the rotor depended from the point in which the bound exist. Using expressions of Equations (1.38) - (1.45) for the rotor response, the slope, bending moment and shearing force are expressed in Equations (1.48) – (1.57).

Slope:

$$S_{y_{ij}}(x,t) = \frac{\partial}{\partial x} \left( Y_{j,\beta}(x,t) \right), \quad S_{z_{ij}}(x,t) = \frac{\partial}{\partial x} \left( Z_{j,\beta}(x,t) \right),$$  \hspace{1cm} (1.48)

$$S_{y_{ij}}(x,t) = \frac{\partial}{\partial x} \left( Z_{j,\beta}(x,t) \right), \quad S_{z_{ij}}(x,t) = \frac{\partial}{\partial x} \left( Z_{j,\beta}(x,t) \right),$$  \hspace{1cm} (1.49)

Bending moment:

$$M_{y_{ij}}(x,t) = EI_j \frac{\partial^2}{\partial x^2} \left( Y_{j,\beta}(x,t) \right) - EI_j \eta \frac{\partial^2}{\partial x^2} \left( Y_{j,\beta}(x,t) \right)$$  \hspace{1cm} (1.50)

$$M_{y_{ij}}(x,t) = EI_j \frac{\partial^2}{\partial x^2} \left( Y_{j,\beta}(x,t) \right) + EI_j \eta \frac{\partial^2}{\partial x^2} \left( Y_{j,\beta}(x,t) \right)$$  \hspace{1cm} (1.51)

$$M_{z_{ij}}(x,t) = EI_j \frac{\partial^2}{\partial x^2} \left( Z_{j,\beta}(x,t) \right) - EI_j \eta \frac{\partial^2}{\partial x^2} \left( Z_{j,\beta}(x,t) \right)$$  \hspace{1cm} (1.52)

$$M_{z_{ij}}(x,t) = EI_j \frac{\partial^2}{\partial x^2} \left( Z_{j,\beta}(x,t) \right) + EI_j \eta \frac{\partial^2}{\partial x^2} \left( Z_{j,\beta}(x,t) \right)$$  \hspace{1cm} (1.53)

Shearing force:

$$V_{y_{ij}}(x,t) = EI_j \left( \frac{\partial^3 Y_{j,\beta}(x,t)}{\partial x^3} - \eta \frac{\partial^3 Y_{j,\beta}(x,t)}{\partial x^3} \right) - \rho A r_{ij} \left( G_{ij,\beta}(x,t) + 2 \Omega G_{ij,\beta}(x,t) \right)$$  \hspace{1cm} (1.54)
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\[
V_{z,s}(x,t) = E L \left( \frac{\partial^3 Y_{z,s}(x,t)}{\partial x^3} + \frac{\partial^3 Y_{s,s}(x,t)}{\partial x^3} \right) - \rho A \frac{r_o^2}{\eta^2} \left( G_{i,s}(x,t) + 2 \Omega \frac{r_o^2}{\eta^2} \frac{r_o^2}{\eta^2} \right) \quad (1.55)
\]

\[
V_{z,s}(x,t) = E L \left( \frac{\partial^3 Z_{z,s}(x,t)}{\partial x^3} - \frac{\partial^3 Z_{s,s}(x,t)}{\partial x^3} \right) - \rho A \frac{r_o^2}{\eta^2} \left( G_{i,s}(x,t) + 2 \Omega \frac{r_o^2}{\eta^2} \frac{r_o^2}{\eta^2} \right) \quad (1.56)
\]

\[
V_{z,s}(x,t) = E L \left( \frac{\partial^3 Z_{z,s}(x,t)}{\partial x^3} + \frac{\partial^3 Z_{s,s}(x,t)}{\partial x^3} \right) - \rho A \frac{r_o^2}{\eta^2} \left( G_{i,s}(x,t) + 2 \Omega \frac{r_o^2}{\eta^2} \frac{r_o^2}{\eta^2} \right) \quad (1.57)
\]

In Equations (1.54) - (1.57), the terms \( g_{i,s} \) express the acceleration and velocity of the slope in a shaft step. The terms are expressed using first-order forward finite differences in the time domain as shown in Equations (1.58) – (1.69).

\[
G_{i,s}(x,t) = \frac{S_{i,s}(x,t) - 2 S_{i,s}(x,t - \Delta t) + S_{i,s}(x,t - 2 \Delta t)}{\Delta t^2} : \text{(Real vertical slope acc.)} \quad (1.58)
\]

\[
G_{i,s}(x,t) = \frac{S_{i,s}(x,t) - 2 S_{i,s}(x,t - \Delta t) + S_{i,s}(x,t - 2 \Delta t)}{\Delta t^2} : \text{(Imag. vertical slope acc.)} \quad (1.59)
\]

\[
G_{i,s}(x,t) = \frac{S_{i,s}(x,t) - S_{i,s}(x,t - \Delta t)}{\Delta t} : \text{(Real vertical slope velocity)} \quad (1.60)
\]

\[
G_{i,s}(x,t) = \frac{S_{i,s}(x,t) - S_{i,s}(x,t - \Delta t)}{\Delta t} : \text{(Imaginary vertical slope velocity)} \quad (1.61)
\]

\[
G_{i,s}(x,t) = \frac{S_{i,s}(x,t) - 2 S_{i,s}(x,t - \Delta t) + S_{i,s}(x,t - 2 \Delta t)}{\Delta t^2} : \text{(Real horiz. slope acc.)} \quad (1.62)
\]

\[
G_{i,s}(x,t) = \frac{S_{i,s}(x,t) - 2 S_{i,s}(x,t - \Delta t) + S_{i,s}(x,t - 2 \Delta t)}{\Delta t^2} : \text{(Img. horizontal slope acc.)} \quad (1.63)
\]

\[
G_{i,s}(x,t) = \frac{S_{i,s}(x,t) - S_{i,s}(x,t - \Delta t)}{\Delta t} : \text{(Real horizontal slope velocity)} \quad (1.64)
\]

\[
G_{i,s}(x,t) = \frac{S_{i,s}(x,t) - S_{i,s}(x,t - \Delta t)}{\Delta t} : \text{(Imaginary horizontal slope velocity)} \quad (1.65)
\]

\[
G_{i,s}(x,t) = \frac{S_{i,s}(x,t) - S_{i,s}(x,t - \Delta t)}{\Delta t} : \text{(Real vertical acceleration)} \quad (1.66)
\]
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\[ G_{y,i}(x,t) = \frac{Y_{y,i}(x,t) - 2Y_{y,i}(x,t - \Delta t) + Y_{y,i}(x,t - 2\Delta t)}{\Delta t^2} : \text{(Imag. vertical acceleration)} \]  
\[ G_{x,i}(x,t) = \frac{Z_{x,i}(x,t) - 2Z_{x,i}(x,t - \Delta t) + Z_{x,i}(x,t - 2\Delta t)}{\Delta t^2} : \text{(Real horiz. acceleration)} \]  
\[ G_{x,i}(x,t) = \frac{Z_{x,i}(x,t) - 2Z_{x,i}(x,t - \Delta t) + Z_{x,i}(x,t - 2\Delta t)}{\Delta t^2} : \text{(Imaginary horizontal acc.)} \]

Before defining the boundary conditions, it is useful to give the rotor static response, which is a result of gravity and of an external constant vertical force \( E_v \). For the simple case of the two-step rotor that will be examined in this chapter, the static response is given for each step as in Equations (1.70), and (1.71) for the case in which both steps have equal radius and equal length \( L_i = L / 2 \).

\[ Y_{s1} = -\frac{\rho A g}{24EI_1} x^4 + \frac{2E_{s1} L - 2E_{x} L_0 + \rho A g L^2}{12EI_1 L} x^3 - \frac{8E_{s1} L^2 L_2 - 12E_{s1} L^3 L + 4E_{s1} L^4 L + \rho A g L^4}{24EI_1 L} x \]  
\[ Y_{s2} = -\frac{\rho A g}{24EI_1} x^4 - \frac{2E_{s1} L - \rho A g L^2}{12EI_1 L} x^3 + \frac{E_{v} L_1}{2EI_1} x^2 - \frac{8E_{s1} L^2 L_2 + 4E_{s1} L^3 L + \rho A g L^4}{24EI_1 L} x^4 + \frac{E_{v} L^3}{6EI_1} \]  

\[ \sigma_i = M_{y,i}(0,t) \]  
\[ \sigma_i = M_{y,i}(L,t) \]  
\[ \sigma_i = F_{y,i} - EI \left( \frac{\partial^3 Y_{s1}}{\partial x^3} \right)_{x=0} - V_{y,i}(0,t) \]
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\[ \sigma_4 = F_{Y_{1,3}} + \frac{\partial^2 Y_{5,1}}{\partial x^2} + M_{Y_{1,3}}(L,t) \]  
(1.75)

Location of disk:

\[ \sigma_5 = M_{Y_{1,3}}(L,t) - M_{Y_{1,3}}(L,t) \]  
(1.76)

\[ \sigma_6 = M_{Y_{1,3}}(L,t) - M_{Y_{1,3}}(L,t) + I_p \Omega G_{2,3}(L,t) + I_y G_{3,3}(L,t) \]  
(1.77)

\[ \sigma_7 = V_{Y_{1,3}}(L,t) - V_{Y_{1,3}}(L,t) - F_{v_y} + E_y + m_y G_{5,3}(L,t) \]  
(1.78)

\[ \sigma_8 = Y_{r,3}(L,t) - Y_{r,3}(L,t) \]  
(1.79)

**Horizontal plane:**

Bearing:

\[ \sigma_9 = M_{Z_{1,3}}(0,t) \]  
(1.80)

\[ \sigma_{10} = M_{Z_{1,3}}(L,t) \]  
(1.81)

\[ \sigma_{11} = F_{Z_{1,3}} - V_{Z_{1,3}}(0,t) \]  
(1.82)

\[ \sigma_{12} = F_{Y_{1,3}} + V_{Z_{1,3}}(L,t) \]  
(1.83)

Location of disk:

\[ \sigma_{13} = S_{Z_{1,3}}(L,t) - S_{Z_{1,3}}(L,t) \]  
(1.84)

\[ \sigma_{14} = M_{Z_{1,3}}(L,t) - M_{Z_{1,3}}(L,t) + I_p \Omega G_{2,3}(L,t) - I_y G_{3,3}(L,t) \]  
(1.85)

\[ \sigma_{15} = V_{Z_{1,3}}(L,t) - V_{Z_{1,3}}(L,t) - F_{v_z} + m_y G_{5,3}(L,t) \]  
(1.86)

\[ \sigma_{16} = Z_{2,3}(L,t) - Z_{1,3}(L,t) \]  
(1.87)

In Equations (1.74), (1.75), (1.82), and (1.83), the values \( F_{v_x} \), \( F_{v_y} \), and \( F_{v_z} \) express the fluid film forces in the vertical and horizontal direction for the bearings on the left \( x = 0 \) and on the right \( x = L \) end as they are defined in Section 3.3, while in Equations (1.78) and (1.86), \( F_{v_y} \) and \( F_{v_z} \) are the vertical and horizontal components of the unbalance force \( F_u = m_u R \Omega^2 \), respectively, with \( F_{v_y} = F_u \cos(\Omega t) \) and \( F_{v_z} = -F_u \sin(\Omega t) \). Note that subscript “R” and “I” in fluid
film forces $F_{ij}$ and $F_{ij}$ declare that the forces have been computed for the real and imaginary response, respectively. Also, the polar and diametrical disk moment of inertia, with the external force, is declared in Equation (1.88).

$$I_p = \frac{1}{2} m_d R_d^2$$
$$I_f = \frac{1}{4} m_d R_d^2 + \frac{1}{12} m_d L_d^2$$
$$E_f = m_d g$$

Here $R_d, L_d, m_d, g$ as the disk radius, disk width, disk mass, and gravity acceleration, respectively.

1.1.2.2 Imaginary Boundary conditions

The boundary conditions $\sigma_i$ for $i = 17, 18, ..., 32$ are formed for both planes in Equations (1.89) - (1.104) and correspond to the bounds of the imaginary properties of the equations of motion.

**Vertical plane:**

Bearings:

$$\sigma_{17} = M_{y_{ij}} (0, t)$$
$$\sigma_{18} = M_{y_{ij}} (L, t)$$

$$\sigma_{19} = F_{y_{ij}} - E I \frac{\partial^3 y_{ij}}{\partial x^3} \bigg|_{x=0} - V_{y_{ij}} (0, t)$$

$$\sigma_{20} = F_{y_{ij}} + E I \frac{\partial^3 y_{ij}}{\partial x^3} \bigg|_{x=L} + V_{y_{ij}} (L, t)$$

**Location of disk:**

$$\sigma_{21} = S_{y_{ij}} (L_i, t) - S_{y_{ij}} (L_1, t)$$

$$\sigma_{22} = M_{y_{ij}} (L_i, t) - M_{y_{ij}} (L_1, t) + I_p \Omega G_{z_{ij}} (L_i, t) + I_f G_{z_{ij}} (L_1, t)$$

$$\sigma_{23} = V_{y_{ij}} (L_i, t) - V_{y_{ij}} (L_1, t) - F_{w_0} + E_f + m_d G_{z_{ij}} (L_1, t)$$
\[ \sigma_{24} = Y_{z,i}(L_1,t) - Y_{i,j}(L_1,t) \]  
(1.96)

**Horizontal plane:**

**Bearings:**

\[ \sigma_{25} = M_{z,j}(0,t) \]  
(1.97)

\[ \sigma_{26} = M_{z,j}(L,t) \]  
(1.98)

\[ \sigma_{27} = F_{z,j} - V_{z,i}(0,t) \]  
(1.99)

\[ \sigma_{28} = F_{i,j} + V_{z,j}(L,t) \]  
(1.100)

**Location of disk:**

\[ \sigma_{29} = S_{z,i}(L_1,t) - S_{z,i}(L_1,t) \]  
(1.101)

\[ \sigma_{30} = M_{z,j}(L_1,t) - M_{z,j}(L_1,t) + I_p \Omega G_{z,j}(L_1,t) - I_f G_{z,j}(L_1,t) \]  
(1.102)

\[ \sigma_{31} = V_{z,j}(L_1,t) - V_{z,j}(L_1,t) - F_{v_1} + m_a G_{z,j}(L_1,t) \]  
(1.103)

\[ \sigma_{32} = Z_{z,j}(L_1,t) - Z_{z,j}(L_1,t) \]  
(1.104)

Note that all boundary conditions are functions of time \( t \).
1.2 Journal bearing support – Worn Bearing

The nonlinear fluid film forces generated by the journal bearing are derived by solution of Reynolds equation which for laminar, isothermal and isoviscous flow is written as in Equation (1.105):

\[
\frac{\partial}{\partial t} \left( \frac{h(\theta)^3}{6\mu} \frac{\partial P_n(\theta, l)}{\partial x} \right) + \frac{1}{R^2} \frac{\partial}{\partial \theta} \left( \frac{h(\theta)^3}{6\mu} \frac{\partial P_n(\theta, l)}{\partial \theta} \right) = \Omega \frac{\partial h(\theta)}{\partial \theta} + 2 \frac{\partial h(\theta)}{\partial t} \tag{1.105}
\]

Reynolds equation is expressed here in terms of the coordinate system \((\theta, l)\), as it is defined in Figure 1.2. In Equation (1.17), the term \(P_n(l, \theta)\) is the pressure developed in the film at time \(t_k\), \(\mu\) is the lubricant viscosity, \(R\) is the journal radius, and \(\theta\) is the angular coordinate (see Figure 2). The fluid film thickness \(h(\theta)\) is defined as in Equation (1.18). The wear angular zone is defined as a function of absolute wear depth \(d_0\) as in [154] in Equations (1.106) and (1.106a). From the starting angle of the wear \((\theta_a)\) up to the ending angle of the wear \((\theta_b)\), the additional relative fluid film thickness is defined in Equation (1.106b).

\[
h(\theta) = 1 + \varepsilon \cos(\theta), \quad 0 \leq \theta \leq \pi
\]

\[
h(\theta) = \begin{cases} 
1 + \varepsilon \cos(\theta - (\phi - \pi)), & \text{for } 0 \leq \theta \leq \theta_a, \theta_a \leq \theta \leq \pi \\
1 + \varepsilon \cos(\theta - (\phi - \pi)) + \delta(\theta), & \text{for } \theta_a < \theta < \theta_b 
\end{cases}
\]

\[
\delta(\theta) = \delta_0 - \left(1 + \cos\left(\theta - \frac{\pi}{2}\right)\right)
\]

\[
\delta(\theta) = \frac{3\pi}{2} - \arccos(1 - \delta_0), \quad \theta_b = \frac{3\pi}{2} + \arccos(1 - \delta_0)
\]

In the previous equations, \(\varepsilon\) is the eccentricity ratio \(\varepsilon = e / c_r\), \(\phi\) is the attitude angle of the journal, \(\dot{e}\) and \(\dot{\phi}\) are the eccentricity ratio and attitude angle velocity, \(c_r\) is the radial clearance, and \(d_0\) and \(\delta_0\) are the absolute and relative wear depth, respectively (see...
Figure 1.2. Note that all angles are defined with respect to the coordinate system in Figure 1.2.

Three loads exist in each journal: a percentage of the weight $W_g$ of the rotor, a percentage of external force $E_F$, and a percentage of the unbalance force $F_u$. The fluid film hydrodynamic reaction in this dynamic load is constituted from the radial $F_r$ and tangential $F_t$ forces, which consequently are functions of time $t$.

The finite difference method is used in this work to solve Reynolds equation. The pressure distribution exists in the half area of the journal, $0 \leq \theta \leq \pi$ and $-L_b/2 \leq \ell \leq L_b/2$ (see Figure 1.2). Since the bearing is assumed to be aligned, Equation (1.105) can be written as in Equation (1.107) in order to express the influence of journal movement in the pressure distribution:
Here $\varepsilon$ is the eccentricity ratio $\varepsilon = e / c_r$, $\phi$ is the attitude angle of the journal, $\dot{\varepsilon}$ and $\dot{\phi}$ are the eccentricity ratio and attitude angle velocity, respectively, and $c_r$ is the radial clearance (see Figure 2.1). In Equation (1.107), each partial derivative is approximated, at any point on the
grid at time \( t \), using central differences expressions with error of second order, by Equations (1.108), (1.109), and (1.110).

\[
\frac{\partial^2 P_{\eta}(l, \theta)}{\partial l^2}_{l=q, \Delta l} = \frac{P_{\eta+1, i} - 2P_{\eta, i} + P_{\eta-1, i}}{\Delta l^2} \quad (1.108)
\]

\[
\frac{\partial^2 P_{\eta}(l, \theta)}{\partial \theta^2}_{l=q, \Delta l} = \frac{P_{\eta, i+1} - 2P_{\eta, i} + P_{\eta, i-1}}{\Delta \theta^2} \quad (1.109)
\]

\[
\frac{\partial P_{\eta}(l, \theta)}{\partial \theta}_{l=q, \Delta l} = \frac{P_{\eta, i+1} - P_{\eta, i-1}}{2\Delta \theta} \quad (1.110)
\]

Variable \( \eta \) is used to express the change of \( l \), variable \( \lambda \) is used to express the change of \( \theta \) and variable \( k \) indicates the change of time \( t \). The first time derivative of \( \varepsilon \) and \( \phi \) are expressed using the forward finite difference expressions \( \dot{\varepsilon} = \left( \varepsilon_{i+1} - \varepsilon_{i} \right) / \Delta t \) and \( \dot{\phi} = \left( \phi_{i+1} - \phi_{i} \right) / \Delta t \). For a given value of time \( t \), \( k = 0, 1, 2, \ldots \), Equation (1.107) is written for \( \lambda = 0, 1, 2, \ldots, \lambda_\nu \), and \( \eta = 0, 1, 2, \ldots, \eta_\nu \), where \( \lambda_\nu - 1 \) and \( \eta_\nu - 1 \) are the number of \( \Delta \theta \) and \( \Delta \lambda \) equal spaces which divide each direction of the pressure domain and the \( (\lambda_\nu + 1) \times (\eta_\nu + 1) \) equations are obtained with respect to variables \( P_{\eta, q, i} \). The \( \lambda_\nu \times \eta_\nu \) mesh is known to be very important for the accuracy of the solution, and in the current analysis a mesh of \( 27 \times 9 \) in the circumferential and axial direction respectively is included as in [180] and [181] as a minimum grid density to extract accurate results. The plain journal bearings are assumed to be open to the atmosphere at the ends \( l = 0 \) and \( l = L_\eta \). With an uncavitated fluid film, the boundary conditions are: \( P_{\eta, 0, i} = P_{\eta, L_\eta, i} = 0 \), which means that \( P_{\eta, 0, i} = P_{\eta, L_\eta, i} = 0 \), \( \lambda = 0, 1, 2, \ldots, \lambda_\nu \). The bearing is also assumed to follow the half Sommerfeld condition (\( \pi \) film bearing) so that \( P_{\eta, (l, \theta)} > 0 \) for \( 0 < \theta < \pi \) and \( P_{\eta, (l, \theta)} = 0 \) for \( \pi \leq \theta \leq 2\pi \). For a given eccentricity \( e \) and attitude angle \( \phi \), the bearing impedance can be calculated by solving the linear \( (\lambda_\nu + 1) \times (\eta_\nu + 1) \) system using an ADI elimination that is suggested for such linear algebraic systems. The impedance forces of the fluid film bearing are calculated in the tangential \( F_t \) and radial \( F_r \) directions (see Figure 1.2) as:
\[ F_r = \sum_{\eta=0}^{\eta_m} \sum_{\lambda=0}^{\lambda_m} \left( P_{\eta,\lambda} \cos(\theta_\eta) R \Delta \theta \Delta \lambda \right) \]

\[ F_i = \sum_{\eta=0}^{\eta_m} \sum_{\lambda=0}^{\lambda_m} \left( P_{\eta,\lambda} \sin(\theta_\eta) R \Delta \theta \Delta \lambda \right) \quad (1.111) \]

The transformation of fluid film forces to the Cartesian coordinates \(Oyz\) is achieved in Equation (1.112):

\[
\begin{bmatrix}
F_x \\
F_y
\end{bmatrix} =
\begin{bmatrix}
\sin(\phi_\lambda) & \cos(\phi_\lambda) \\
-\cos(\phi_\lambda) & \sin(\phi_\lambda)
\end{bmatrix}
\begin{bmatrix}
F_r \\
F_i
\end{bmatrix}
\quad (1.112)
1.3. Rotor bearing system construction

In this section, we propose an approach that makes the current analysis quite different from the rest of the literature: the journal is not treated as a rigid cylindrical part of the rotor included in the bearing, but as a continuous and deformable extension of the entire shaft, which has dynamic characteristics that are a result of the entire model dynamic response. Up to now, there have been very precise investigations as to the journal mobility, and generally in the dynamics of the combination of the journal and the bearing, but this analysis uses a very precise model for the continuous shaft; every dynamic characteristic that can affect the journal mobility is considered, such as the shearing force in every point of the shaft, which is a significant parameter in the journal mobility, especially when the rotor bearing system passes through a critical speed. The combination of those two machine elements (shaft and bearing) results in a quite complex and strongly nonlinear system of equations that describes the entire rotor bearing system efficiently, even at rotating speeds very close to the critical speed.

In Section 1.2, there are four variables used as inputs for the calculation of the bearing impedance force, and they must be expressed as functions of the rotor dynamic response. In Equation (1.107), the eccentricity and the attitude angle of each journal together with the respective velocities are expressed as functions of \( Y_j(x,t) \) and \( Z_j(x,t) \) for time \( t \), as in the following:

\[
\begin{align*}
\epsilon_{1,j} &= \sqrt{(Y_j(0,t)) + (Z_j(0,t))}, \quad \dot{\epsilon}_{1,j} = \frac{\epsilon_{1,j} - \epsilon_{1,j-\Delta}}{\Delta} \quad \phi_{1,j} = \arctan\left(\frac{Y_j(0,t)}{Z_j(0,t)}\right) \quad (1.117) \\
\dot{\phi}_{1,j} &= \frac{Z_j(0,t) \left( Y_j(0,t) - Y_j(0,t-\Delta) \right) - Y_j(0,t) \left( Z_j(0,t) - Z_j(0,t-\Delta) \right)}{\Delta \left( \epsilon_{1,j} \right)^2} \quad (1.118) \\
\epsilon_{2,j} &= \sqrt{(Y_j(0,t)) + (Z_j(0,t))}, \quad \dot{\epsilon}_{2,j} = \frac{\epsilon_{2,j} - \epsilon_{2,j-\Delta}}{\Delta} \quad \phi_{2,j} = \arctan\left(\frac{Y_j(0,t)}{Z_j(0,t)}\right) \quad (1.119) \\
\dot{\phi}_{2,j} &= \frac{Z_j(0,t) \left( Y_j(0,t) - Y_j(0,t-\Delta) \right) - Y_j(0,t) \left( Z_j(0,t) - Z_j(0,t-\Delta) \right)}{\Delta \left( \epsilon_{2,j} \right)^2} \quad (1.120)
\end{align*}
\]
From the above equations, it is clear that the pressure deviation in each bearing is a function of the constants \( q_i, i = 1, 2, ..., 32 \). In this way, a system of 32 equations is contained in Equation (1.121). Note that each variable in Equation (1.107) is expressed twice, once for a real property and once for an imaginary property, but in the above Equations (1.117) – (1.120), they are declared no matter what property they express (real or imaginary). For example, the magnitude \( F_{Y_{i,0}} \) in the boundary condition of Equation (1.74) is a result of using \( e_{Y_{i,0}} = \sqrt{(Y_{i,0}(0, t_i))^2 + (Z_{i,0}(0, t_i))^2} \), but magnitude \( F_{Y_{i,0}} \) in Equation (1.91) is a result of using \( e_{Y_{i,0}} = \sqrt{(Y_{i,0}(0, t_i))^2 + (Z_{i,0}(0, t_i))^2} \).
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\[ \sigma_i(q_1, q_2, ..., q_{32}) = 0, \quad i = 1, 2, ..., 32 \]  \hspace{1cm} (1.121)

This system is solved using the Newton-Raphson (N-R) method combined with a system parameter modification that offers convergence with a random initial guess. The initial value is a significant parameter in the entire progression of the solution, and with the parameter modification method, the initial values locate both journals in positions very close to the equilibrium positions that the journals would occupy if no dynamic loading existed. To explain further, each equation of the system in Equation (1.121),

\[ \sigma_i(q_1, q_2, ..., q_{32}) = 0, \quad i = 1, 2, ..., 32 \]

is written in the form of Equation (1.122).

\[ \sigma_i(q_1, q_2, ..., q_{32}) - C_i = 0, i = 1, 2, ..., 32 \]  \hspace{1cm} (1.122)

Constant \( C_i \) is given as: \( C_i = \sigma_i(q_1(0), q_2(0), ..., q_{32}(0)), i = 1, 2, ..., 32 \).

The system in Equation (1.122) can be now expressed as in Equation (1.123).

\[
\begin{align*}
\sigma_1' &= \sigma_1 - \frac{N - K}{K} C_1 = 0 \\
\sigma_2' &= \sigma_2 - \frac{N - K}{K} C_2 = 0 \\
\vdots \\
\sigma_{32}' &= \sigma_{32} - \frac{N - K}{K} C_{32} = 0 
\end{align*}
\]  \hspace{1cm} (1.123)

Term \( K \) is the number of iterations that are needed for the system in Equation (1.123) to become equivalent to the initial system in Equation (1.121), and \( N \) is the number iterations with \( N = 1, 2, ..., K \). It is clear that when \( N = K \), the system in Equation (1.123) is equivalent to the system in Equation (1.121), which means \( \sigma_i' = \sigma_i, i = 1, 2, ..., 32 \). The number \( K \) is dependent on the sensitivity of the system equations to each variable difference. The initial guess for \( q_i(0), i = 1, 2, ..., 32 \) must be different from zero so that values in Equations (1.117) to (1.120) are definable.
For every value of \( N \), the N-R method is used to obtain a solution of each system \( \sigma_i^N \). The Jacobian matrix \( J \) in Equation (1.124) must be determined numerically because the fluid film forces do not have analytical expressions with respect to the unknown variables. The Jacobian matrix in Equation (1.124) is determined by making a perturbation \( \Delta q \) in each one of the unknown variables, and in such way each partial numerical derivative \( \frac{\partial \sigma_i^N}{\partial q_j(t_i)} \) is determined. For example:

\[
\frac{\partial \sigma_i^N}{\partial q_j(t_i)} = \frac{\sigma_i^N(q_j(t_i) + \Delta q, q_2(t_i), \ldots, q_{12}(t_i)) - \sigma_i^N(q_j(t_i), q_2(t_i), \ldots, q_{12}(t_i))}{\Delta q}
\]

The Jacobian matrix of each iteration is formed as in Equation (1.124).

\[
J_i = \begin{pmatrix}
\frac{\partial \sigma_1^N}{\partial q_1(t_1)} & \frac{\partial \sigma_1^N}{\partial q_2(t_1)} & \cdots & \frac{\partial \sigma_1^N}{\partial q_{12}(t_1)} \\
\frac{\partial \sigma_2^N}{\partial q_1(t_1)} & \frac{\partial \sigma_2^N}{\partial q_2(t_1)} & \cdots & \frac{\partial \sigma_2^N}{\partial q_{12}(t_1)} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial \sigma_{12}^N}{\partial q_1(t_1)} & \frac{\partial \sigma_{12}^N}{\partial q_2(t_1)} & \cdots & \frac{\partial \sigma_{12}^N}{\partial q_{12}(t_1)}
\end{pmatrix}
\]

The solution of the system in Equation (1.123) is achieved following the N-R method as in Equation (1.125) with \( i \) indicating the number of N-R iteration.

\[
\begin{pmatrix}
q_i(t_1) \\
q_i(t_2) \\
\vdots \\
q_i(t_{12})
\end{pmatrix} \quad \begin{pmatrix}
\frac{\partial \sigma_1^N}{\partial q_1(t_1)} & \frac{\partial \sigma_1^N}{\partial q_2(t_1)} & \cdots & \frac{\partial \sigma_1^N}{\partial q_{12}(t_1)} \\
\frac{\partial \sigma_2^N}{\partial q_1(t_1)} & \frac{\partial \sigma_2^N}{\partial q_2(t_1)} & \cdots & \frac{\partial \sigma_2^N}{\partial q_{12}(t_1)} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial \sigma_{12}^N}{\partial q_1(t_1)} & \frac{\partial \sigma_{12}^N}{\partial q_2(t_1)} & \cdots & \frac{\partial \sigma_{12}^N}{\partial q_{12}(t_1)}
\end{pmatrix}^{-1} \quad \begin{pmatrix}
\sigma_1^N(q_1(t_1), q_2(t_1), \ldots, q_{12}(t_1)) \\
\sigma_2^N(q_1(t_1), q_2(t_1), \ldots, q_{12}(t_1)) \\
\vdots \\
\sigma_{12}^N(q_1(t_1), q_2(t_1), \ldots, q_{12}(t_1))
\end{pmatrix}
\]
In this way, the response of the system is calculated for every time \( t_k \), and as shown in the following numerical example, the system response is feasible even at high dynamic loads near the critical speed where the shearing force obtains values that do not only depend on the external loads or gravity.

It is a fact that as shown in current section the utility approach adopted in the modeling of the rotor system is very limited since it concerns only one disc and a crack and leads in quite complex equations even if the crack and the disk are located at the same place. The expansion of the model increases dramatically for every rotor step that is added (plus 16 rows in Equation (1.125) for every additional step) that results in a quite time consuming evaluation of the Jacobean matrix \( J \) in Equation (1.124). The ability of introducing a disk, a crack, or a bearing demands an additional rotor step. For example, in order to locate a crack in a different point from this of the disc a three step rotor is demanded. But it is not the equations of motion that increases this great time of evaluation duration; the finite bearing formulation and the finite difference method is the main reason that the evaluation delays. Consider that for a 32X32 Jacobean 1024 evaluations of each bearing are demanded and this just for one N-R step. The 32X32 system yields a start up of Figure 1.3a in almost 100 hrs using an Intel Core 2 duo® Pentium® 3.4 GHz processor. As it was investigated, the fact of substituting the finite bearings with the quite simple analytical expressions of Short/Long bearings yields the same result in just 1.5 hrs with almost equal results adopting of course a finite bearing with low or high L/D ratio. This encouraging result prompts the efficient simulation of real multi-stepped rotor bearing systems that include short or long fluid bearings. On the other hand if the bearings have to be modeled as “finite” a parallel processing can decrease the evaluation time in much lower tasks.

Some comments about two simulation code parameters must be made in order to make clear some matters that have with the correspondence of code in real effects to do. A parameter that must be highlighted in simulation code is the unbalance force. As it is also shown in section 1.1.2, the unbalance force is defined as:

\[
F_u = m_u \left( R_u + e_u + \sqrt{Y_{1,a}^2(l,t)^2 + Z_{1,a}^2(l,t)^2} \right) \Omega^2; \quad \text{making the unbalance force a function of}
\]

unbalance mass \( m_u \) (experimentally calculated during balancing (see chapter 4, section 4.4)), rotor bowing at disk plane \( e_u \) (experimentally measured), elastic rotor response magnitude at disk-unbalance plane \( \sqrt{Y_{1,a}^2(l,t)^2 + Z_{1,a}^2(l,t)^2} \) and rotational speed \( \Omega \). The unbalance force
coexists with the external load due to disk mass $F_E$ as it is also shown in section 1.1.2. Note also that the unbalance force is transformed in every time step in vertical and horizontal component and thus is added in boundary conditions for disk location point as it is shown in section 1.1.2.

The entire progress (flow) of the algorithm that is developed for the calculation of system response is shown schematically in the next figure so as to declare in a more compact and understandable way what is behind the word “algorithm” since from now and on it will be pronounced often.

**Algorithm flow**

![Algorithm flow diagram](image)

Another parameter is the material loss factor that is set to a value just to “cut out” the infinite response in order to make the start-up computable, and to fit better the experimental system amplitude in critical speed. In this work, a variable loss factor is not considered because the internal damping is treated as a technique in order to avoid the infinite response that cannot
be damped by the bearing damping coefficients. Actually it is a very difficult matter to incorporate variable loss factor that would be a function of stresses and this is because the variable loss factor demands different solution technique of equations of motion using hyper geometric series. This concept is left out from current work but is included in future research.

1.4 Numerical Application 1

The results include the rotor response time history for a virtual start up of the rotor bearing system of Figure 1.1. Physical and geometric properties are given in Table 1.1 for a shaft of Stahl 37.

<table>
<thead>
<tr>
<th>Physical Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaft Radius R</td>
<td>0.025 m</td>
</tr>
<tr>
<td>1st Step Length L₁</td>
<td>0.523 m</td>
</tr>
<tr>
<td>2nd Step Length L₂</td>
<td>0.422 m</td>
</tr>
<tr>
<td>Disk Radius R₃</td>
<td>0.19 m</td>
</tr>
<tr>
<td>Disk Density ρ</td>
<td>7832 kg/m³</td>
</tr>
<tr>
<td>Disk Width L₄</td>
<td>0.022 m</td>
</tr>
<tr>
<td>Oil Viscosity μ</td>
<td>0.002 Pa s</td>
</tr>
<tr>
<td>Young Mod. E</td>
<td>2.068 GPa</td>
</tr>
<tr>
<td>Shaft Density ρ</td>
<td>7832 kg/m³</td>
</tr>
<tr>
<td>Bearing Length Lᵃ</td>
<td>0.025 m</td>
</tr>
<tr>
<td>Bear. Radial Clear. Cr</td>
<td>40 μm</td>
</tr>
</tbody>
</table>

Table 1.1 Geometric and physical properties of the current rotor bearing system

The system start up is performed from the initial rotational speed \( Ω = 50 \text{ rad/s} \) to the maximum \( Ω = 450 \text{ rad/s} \) with an acceleration of \( \dot{Ω} = 40 \text{ rad/s}^2 \), while the sampling frequency is \( 1/\text{dt} = 800 \text{ Sam./s} \) (see Figure 1.3). Each time value corresponds in a different rotational speed since the start up is continuous and in a time range of 100 s the system passes through the rotational speed range of 50 to 450 rad/s.
Chapter 1 – A non linear, dynamic, continuous, damped model for rotor bearing system oscillations

Figure 1.3 Time history during start up for a) Journal 1 vertical, b) Mid-span vertical, c) journal 1 horizontal and d) Mid-span horizontal response.

Figure 1.4 Frequency response during start up (100 sec) in a) journal 1 and b) mid-span.

Figure 1.5 Rotor orbits during start up in a) journal 1 and b) mid-span.
Next, the frequency response is extracted from the above signals of Figure 1.3 and plotted in Figure 1.4. Also see Figure 1.5 for rotor orbits.

A time-frequency analysis using Short Time Fourier Transform is made in order to inspect the development of higher (or lower) harmonics due to high non-linearity of the system. Note that a Gaussian noise of 20dB Signal to Noise Ratio is added in the resulting time histories. The non-linearity of the system could be easily recognized through the terms of the system Equations (1.117)-(1.120) where the feedback of the response in the solution is expressed. As it is shown in Figure 1.6 higher harmonics are developed due to the non-linear fluid film forces with their magnitude to be increased as the system goes towards the resonance. Note that the introduction of internal damping is here a tool for cutting the theoretical infinity response and is considered constant (not depended of stresses) while evaluation.

![Figure 1.6 Logarithmic specgram representation of for the time histories of a) journal 1 and b) mid-span response in vertical plane.](image)

The following remarks could be made by the current analysis;

1) The current model provides a different assumption for bearing impedance as a function of rotor response resulting dynamic properties that with other assumption (linear) could be hidden.

2) The nonlinear dynamical system can be accurately solved, with the bearing forces to be obtained using finite differences method without using stiffness and damping coefficients as it has been widely applied in literature.

3) With the resulting time histories of the system response and using the time-frequency decomposition additional system harmonics are inspected in various frequencies of operation.
4) The solution is feasible even in critical speeds with the bearing impedance to become computable without the need of equilibrium position consideration.

### 1.5 Numerical Example 2

In this numerical example a rotor bearing system from the literature is analyzed using the current mathematical model. The specifications consist with the Lund’s Rotor. Consider a rotor bearing system as in Figure 1.7 with the characteristics of Table 1.2 (material: Stahl 37).

<table>
<thead>
<tr>
<th>Rotor length:</th>
<th>$L = 1.27 , m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor radius:</td>
<td>$R = 0.0508 , m$</td>
</tr>
<tr>
<td>Material density:</td>
<td>$\rho = 7832.06 , kg/, m^3$</td>
</tr>
<tr>
<td>Young Modulus:</td>
<td>$E = 206.85 , GPa$</td>
</tr>
<tr>
<td>Bearing Length:</td>
<td>$L_b = 0.0254 , m$</td>
</tr>
<tr>
<td>Bearing Radial Clearance:</td>
<td>$c_r = 50.8 , \mu m$</td>
</tr>
<tr>
<td>Lubricant viscosity:</td>
<td>$\mu = 0.014 , Pa, Sec$</td>
</tr>
<tr>
<td>External Force:</td>
<td>$EF = 0., Nt$</td>
</tr>
</tbody>
</table>

**Table 1.2. Physical and geometric properties of Lund’s Rotor.**

Both bearings are located at the ends of the shaft having the same characteristics, and the shaft is considered as homogenous and of constant cross section. The excitation is an unbalance provoked from a mass $m_u = 0.001 \, kg$ located in the mid-span in a radius $R_u = 0.05 \, m$.

The excitation as long as the vertical constant external force $EF$ require a two step rotor model in order to be imported to the boundary conditions but in practice the rotor is constituted from one step. Note that in current example axial torque and external force are set equal to zero $T = 0$ and $EF = 0$.

*Figure 1.7 The Lund’s Rotor in finite bearings*
The rotor model starts up from an initial rotational speed $\Omega = 100 \text{ rad/s}$ up to the final speed of $\Omega = 1000 \text{ rad/s}$. The rotor response is calculated in each rotational speed, using a time step $\Delta t = (2\pi / 100\Omega) s$, in three points: the left bearing (journal 1 response), the right bearing (journal 2 response) and the rotor’s mid-span. The rigid bearing critical speed of the rotor is calculated equal to $\Omega_{\text{CR}} = 799.6 \text{ rad/s}$ so the system passes through the first critical speed.

Both journal responses are shown in Figure 1.8 and as it is expected due to symmetry both journals have responses equal to each other.

The rotor bearing system runs at four different speeds $\Omega = 1500, 2500, 3500, 4500 \text{ rpm}$ under the first critical and the journal responses are shown in Figure 1.8 comparing the cases of non worn bearings and 50% worn bearings. Also the stiffness and damping coefficients are calculated for this range of rotational speeds for both cases and they are shown in Figures 1.9 and 10. The calculation of linear stiffness and damping coefficients require very low journal vibration amplitude (usually below $c_j / 1000$) and this is feasible only in rotational speeds far from critical speeds even with small unbalance.

![Figure 1.8](image-url)

*Figure 1.8. a) left and b) right journal equilibrium positions for non worn and 50% worn bearings and for $L/D=0.5$.***
The stiffness and damping coefficients have an obvious change when the bearings are worn. As it shown in Figures 1.9 and 1.10 the direct stiffness has a decrement when the bearing is worn but this happens only for a specific rotational speed range. The change in the coefficients is depended as shown not only from the wear percentage but also from the rotational speed.

In Figure 1.11 the response of journal 1 (left journal) is calculated for variable rotational speeds before and after first critical speed. It is shown that there are speeds that provoke journal trajectories around an equilibrium positions and speeds in which journal trajectories...
become with progress near radial clearance when the rotor speed become closer to the critical speed.

As mentioned above the non-linearity of fluid film forces, in combination with the existence of the crack and the wear, assign in rotors response harmonics that each of them can be amplified depending on the defect and the rotational speed. In the case of non-linear modeling even without the presence of any defect the numerical results show that the stability of the system varies with the speed magnitude and also that the dynamic behavior of the system include 2T-periodic, quasi-periodic and chaotic motions. Many properties and techniques have been used to investigate the existence of chaotic motions-chaotic attractors. A general view of changing from periodic motions to chaotic behavior can be given by Fourier transforms and bifurcation diagrams but both methods cannot distinguish the chaotic from the random phenomena. The measures that are used to identify chaotic systems are fractal dimension and there are many definitions for it additionally with the Lyapunov exponent. The
most widely used definition for fractal dimension is based on the correlation dimension $d_G$ offering high computational speed. The Lyapunov exponent can be used for a conservative and a non-conservative, as the current one, system but its use for experimental data is difficult. The fractal dimension on the other hand can be used only for non-conservative systems and is easy used in experimental data.

Poincaré maps are used in this analysis to identify the inclusion of aperiodic or chaotic motion in the dynamic system flow. A point on the Poincaré section can be referred as the point of the trajectory (Z-Y plane) in which the system will return after a time amount of $T$, where $T$ is the period of the driving excitation force. The projection of a Poincaré section on the $Z(nT)-Y(nT)$ plane is referred as the Poincaré map of the system.

Figure 1.12. Poincaré maps of Journal 1 time response for rotational speeds of a)100, b) 300, c) 500, d) 800, e) 850, f) 900, g) 1000, h) 1100, i) 1200 rad/s for L/D=0.5.
Note that the Poincaré maps have been consisted from time series obtained by a time interval 
\[ \Delta t = (T/45) \] s with solutions of first 300-500 periods to be rejected (depended to the situation) so as to achieve steady state solutions. The time series length was about 30000 samples except the cases that system instability was presented.

When the rotational speed is low with respect to the critical speed the journal trajectories describe a motion around an equilibrium position as it is shown in Figure 1.11. When the rotational speed increases further approaching critical speed then the journal is forced to motions near limit cycle due to the large amplitudes that begin to appear as it is shown in Figure 1.11 f-h. Increasing further the rotational speed after the critical the journals return to trajectories around an equilibrium positions as it is shown in Figure 1.11 i-l. In Figure 1.11 l is shown that there is an initial trajectory around an equilibrium position but due to the fact that the motion is developed near the bearing center the instability that characterizes this special point forces the journals to motions near limit cycle.

The fact that the fluid film forces are strongly non-linear, effect the rotor motion by introducing asynchronous harmonics to the driving force frequency. This fact can be observed in the cases that the fluid film forces can be characterized as non linear and this is depended in the rotational speed and unbalance force. Using the current algorithm the non-linearity in fluid film forces is a property that is being adopted in the system when the rotor journals are forced to execute motions far from equilibrium positions. As it is shown in Figure 1.13a and Figure 1.14a the journal 1 and mid-span time series does not contain any harmonic except from synchronous, but this happens in a low rotational speed of 100 rad/s that can not produce high unbalance-excitation force. The same phenomenon is observed also in a bit higher rotational speed of 200 rad/s as it is shown in Figure 1.13 and Figure 1.14b. When the rotational speed become 400 rad/s it is clear that the rotor mid-span as long as the journal 1 time series contain higher harmonics introduced by the fluid film non-linear forces, see Fig 1.13c and Figure 1.13c. In this example only the second harmonic is amplified but if the unbalance force becomes greater, then higher harmonics are also amplified as shown in Figures 1.13e,f and 1.14e,f.
Figure 1.13 FFT of journal 1 response in vertical plane for a) 100 rad/s, b) 200 rad/s, c) 400 rad/s, d) 500 rad/s, e) 900 rad/s, f) 1100 rad/s.

Figure 1.14 FFT of mid-span response in vertical plane for a) 100 rad/s, b) 200 rad/s, c) 400 rad/s, d) 500 rad/s, e) 900 rad/s, f) 1100 rad/s.
1.6 Conclusions

The main target of this chapter was to present a new model of construction of a rotor bearing system combining a continuous Rayleigh shaft with finite fluid film bearings in such a way that no bearing coefficients are used, as in most research works up to now, so as to achieve an accuracy even with unpredictable rotor-journal motions in any location of the radial clearance area.

Since the current system belongs to those called “dynamic systems” there was a need to investigate the progress of this attractor (system of equations) so as to have an idea of the kind of motions that can be developed from such high non-linear systems being strongly depended from initial conditions.

From the current analysis there can be made some useful conclusions especially in the effect of worn fluid film bearings in the shaft dynamic.

1) The non-linearity in fluid film forces causes some interesting results such as higher harmonics of frequency 2xrev or 3xrev even in low operation rotational speeds where the non-linearity is not intense.

2) Higher harmonics of any multiple of synchronous are observed in rotor motion of the system when the non-linearity becomes more intense in cases such as great unbalance without to be necessarily combined with high rotational speeds.

3) The rotor is forced to execute motions in the entire area of radial clearance when the rotational speeds get near to critical and this fact is due to the developed shearing forces that in the current model are combined with the fluid film impedance.

4) Chaotic motions were not observed in the rotational speeds analyzed. Such motions can probably be presented very close to critical speeds where the fluid film forces have to confront the violent rotor motion.

5) The rotor motions are characterized periodic and quasi periodic depending in the fluid film forces non-linearity.

6) Periodic motions with period of $1T$ are developed in low operational speeds. In higher speeds the motion period becomes $2T$ or even $8T$ in special cases before becoming quasi-periodic in further speed increment.

The current dynamic system is characterized by an adaptability in linear and non-linear behavior depending in the fluid film impedance without any special adjustment in bearing
properties since the way that the bearing are introduced in the shaft can confront to any situation that the linear bearing consideration is judged insufficient.
Chapter 2  
Breathing Crack Compliance Matrix Calculation

A local compliance matrix of two degrees of freedom, bending in the horizontal and the vertical planes is used to model the rotating transverse crack in the shaft and is calculated based on the available expressions of the stress intensity factors and the associated expressions for the strain energy release rates.

The compliance matrix is calculated for the first time at any angle of rotation. Thus, the compliance is given as a function of both the crack depth and the angular location. These expressions are usable, due to the stress intensity function limitations, only for limited regions around the zero angular position of the crack and not for every crack angle. For these cases, B-spline curves are used to interpolate the known points and a function in analytical form is given for every crack depth and angle. It is well known that when a crack exists in a structure, such as a beam, then the natural frequency of vibration decreases. This reduction is studied here for six independent parameters namely the depth, the location, and the rotational angle of each crack. By keeping these six parameters constant, the first three flexural eigenmodes can be computed and plotted.

2.1 Model of the stationary transverse crack

The local compliance method is used here to model the crack. The material of the shaft is considered to be homogeneous and isotropic with Young modulus of Elasticity $E$ and Poisson ratio $v$. The radius of the cross section is $R$ and the shaft is subjected to loads $P$. The crack depth is $a$. The crack can be bounded in the $x$ direction by $-b$ and $b$ and in the $y$ direction by $0$ and $a$, (see Figure 2.1). The boundary $b$ can be calculated using Pythagorean Theorem from Figure 2.1, $b = \sqrt{R^2 - (R-a)^2}$ and the dimensionless form, by defining $\bar{b} = b / R$ and $\bar{a} = a / R$ is $\bar{b} = \sqrt{1 - (1 - \bar{a})^2}$.
The cracked strip of finite width $dx$ and height $h_i$ has a crack depth $a_i$. From the geometry the following equation $h_i = 2\sqrt{R^2 - x^2}$ is obtained. If $\pi = x / R$ and $\eta_i = h_i / R$, then $\eta_i = 2\sqrt{1 - \pi^2}$ in dimensionless form. If $a_i = \frac{h_i}{2} - (R - a)$ or $a_i = \frac{h_i}{2} - (R - a) = \sqrt{R^2 - x^2} - (R - a)$, $\bar{a}_i = a_i / R$ and $\bar{a} = a / R$ then the dimensionless crack depth is given by:

$$\bar{a}_i = \frac{\eta_i}{2R} - \left(1 - \frac{a}{R}\right) = \sqrt{1 - \pi^2} - (1 - \bar{a})$$

### 2.1.1 The direct compliance calculation

The strain energy density function is defined as $J(y)$ and the bending moments in the two main directions are defined as $P_5$ for the vertical plane, and as $P_4$ for the horizontal plane.

$$J(y) = \frac{1}{E} \left[ \left( K_{ii} \right)^2 + \left( K_{ii} \right)^2 + m \left( K_{mii} \right)^2 \right]$$  \hspace{1cm} (2.1)

$$K_{ii} = \sigma_5 \sqrt{\pi} F_2 \left( \frac{\eta_i}{h_i} \right), \ K_{mii} = K_{mii} = 0, \ \sigma_5 = \frac{4P_5 \sqrt{R^4 - x^2}}{\pi R^4}$$  \hspace{1cm} (2.2)

$$K_{ii} = \sigma_4 \sqrt{\pi} F_1 \left( \frac{\eta}{h} \right), \ K_{mii} = K_{mii} = 0, \ \sigma_4 = \frac{4P_4 x}{\pi R^4}$$  \hspace{1cm} (2.3)
Chapter 2 – Breathing crack compliance matrix calculation

\[ F_1 \left( \frac{y}{h} \right) = F_0 \left( \frac{y}{h} \right) \left[ 0.752 + 2.02 \frac{y}{h} + 0.37 \left( 1 - \sin \frac{\pi y}{2h} \right)^3 \right] \], \quad (2.4)

\[ F_2 \left( \frac{y}{h} \right) = F_0 \left( \frac{y}{h} \right) \left[ 0.923 + 0.199 \left( 1 - \sin \frac{2\pi y}{2h} \right)^4 \right], \quad E' = \frac{E}{1 - \nu^2} \] (2.5)

\[ F_0 \left( \frac{y}{h} \right) = \sqrt{\tan \frac{\pi y}{2h} / \frac{\pi y}{2h} / \cos \frac{\pi y}{2h}} \] (2.6)

In an orthogonal cross section of unitary width the additional transposition in direction 5 (vertical plane) because of the crack of depth \( a \) is:

\[ U_5 = \frac{\partial}{\partial P_5} \left[ \int_{-a}^{a} J(y) dy \right], \quad U'_5 = \frac{\partial}{\partial P'_5} \left[ \int_{-a}^{a} J(y) dy \right] \] (2.7)

Then, the compliance because of the crack per width unit is \( C_{55}' = \frac{\partial U_5}{\partial P_5} \), \( C_{44}' = \frac{\partial U_4}{\partial P_4} \) and after integration the local compliance is obtained.

\[ C_{55} = \frac{\partial^2}{\partial P_5^2} \left( \int_{-a}^{a} J(y) dy dx \right), \quad C_{44} = \frac{\partial^2}{\partial P_4^2} \left( \int_{-b}^{b} J(y) dy dx \right) \] (2.8)

In order to obtain dimensionless relations,

\[ \bar{x} = \frac{x}{R}, \quad \bar{y} = \frac{y}{R}, \quad \bar{a} = \frac{a}{R}, \quad \bar{b} = \frac{b}{R} \] and \( \bar{h} = \frac{y}{h} = \frac{y}{R} \left( 2\sqrt{R^2 - x^2} \right) = \frac{\bar{y}}{2\sqrt{1 - \bar{x}^2}} \), \( b = \sqrt{1 - (1 - a)^2} = \sqrt{2a - a^2} \).

The integrations in Equations (8) are bounded, for \( x \) from \(-\bar{b}\) to \( \bar{b} \) and for \( \bar{y} \) from 0 to \( a, = \sqrt{1 - \bar{x}^2} - (R - a) \).

As it is obvious from previous definitions the boundaries of integration depend on value \( \bar{x} \).

Then

\[ \bar{C}_{55} = c_{55} \frac{ER}{1 - \nu^2} = \frac{32}{\pi} \int_{\frac{\pi}{2}}^{\pi} d\pi x^2 \left( 1 - \pi^2 \right) F_0^2 \left( \frac{\bar{y}}{\bar{h}} \right) \] (2.9)

\[ \bar{C}_{44} = c_{44} \frac{ER}{1 - \nu^2} = \frac{16}{\pi} \int_{\frac{\pi}{2}}^{\pi} d\pi y^2 \left( \frac{\bar{y}}{\bar{h}} \right) \] (2.10)

Integration on the cracked surface gives the values of compliance Table 2.1 and the diagram of the dimensionless compliance \( \bar{C}_{55} \) as function of the crack depth is shown in Figure 2.3.
Figure 2.3 (a) The dimensionless compliance $C_{55}$ and (b) $C_{44}$ as function of the crack depth.

<table>
<thead>
<tr>
<th>$\alpha / R$</th>
<th>$C_{55}$</th>
<th>$C_{44}$</th>
<th>$\alpha / R$</th>
<th>$C_{55}$</th>
<th>$C_{44}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.04</td>
<td>0.002973</td>
<td>0.00035</td>
<td>0.56</td>
<td>1.600490</td>
<td>0.378117</td>
</tr>
<tr>
<td>0.08</td>
<td>0.016190</td>
<td>0.000391</td>
<td>0.60</td>
<td>1.892200</td>
<td>0.494023</td>
</tr>
<tr>
<td>0.12</td>
<td>0.043041</td>
<td>0.001606</td>
<td>0.64</td>
<td>2.219100</td>
<td>0.638242</td>
</tr>
<tr>
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<td>0.085426</td>
<td>0.004368</td>
<td>0.68</td>
<td>2.585270</td>
<td>0.817164</td>
</tr>
<tr>
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<td>0.009499</td>
<td>0.72</td>
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<td>1.038880</td>
</tr>
<tr>
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<td>1.313720</td>
</tr>
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<td>2.614330</td>
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<td>3.288390</td>
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<td>0.153949</td>
<td>0.96</td>
<td>6.815160</td>
<td>4.147720</td>
</tr>
<tr>
<td>0.48</td>
<td>1.109450</td>
<td>0.218119</td>
<td>1.00</td>
<td>7.790390</td>
<td>5.266350</td>
</tr>
<tr>
<td>0.52</td>
<td>1.340520</td>
<td>0.285422</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.1 Dimensionless values of the local compliance function.
2.1.2 The direct compliance of the rotating crack

This is an important issue for the behavior of the crack for the respective model. The crack opens and closes during the rotation of the shaft according to the angle of rotation, if the displacement due to the weight of the shaft dominates on the vibration amplitude. This is the case considered here, a case that is very common in turbines and generators. The local compliance due to the crack is a periodic function of the angle of rotation. There is, however, the case where the vibration amplitude is equal or greater of the displacement due to the gravity. In this case the problem is non-linear and the opening-closing of the crack depends on the direction of the moment component along the crack direction.

In the case that the crack lies in a rotated arbitrary position in respect with the two main axes by following the same method as in the previous paragraph, the new compliances in the two main directions may be calculated. The direction of the bending load divides the cross section of the shaft into two areas, one of tension and one of compression. The crack may be entirely located in one of these two areas and in this case is loaded in tension or in compression. It could, however, be located in both areas as in Figure 2.4 and thus the part of the crack which is located in the tension area is the opening part and the part in the compression area is the closing part. Only the part in tension gives additional flexibility to the shaft and thus the part in compression should be omitted from the compliance calculation.

![Diagram showing areas of tension and compression](https://via.placeholder.com/150)

*Figure 2.4 The area of tension and compression in the cracked section with rotated crack of small depth.*
A Cartesian coordinate system \( XOY \) is considered Figure 2.5, with point \( O \) to be in the centre of the circular cross section. The angle \( \phi \) defines the rotational position of the crack and it takes values from 0 to 180 degrees in order to cover all the rotational situations. The bounds for the integration of Equations (2.9) and (2.10) are now \(-b_1\) to \(-b_2\) for the variable \( x \), and 0 to \( a_s \) for the variable \( y \). The above mentioned quantities are defined as follows.

\[
b_1 = b \cos \phi - s, \quad b_2 = b \cos \phi + s \quad \text{and} \quad s = (R - a) \sin \phi
\]

\[
b_1 = b \cos \phi - (R - a) \sin \phi, \quad b_2 = b \cos \phi + (R - a) \sin \phi
\]

\[
y_1 = b \sin \phi + (R - a) \cos \phi, \quad y_2 = -b \sin \phi + (R - a) \cos \phi
\]

\[
a_s = h_r / 2 - y_2 - k \quad \text{Where} \quad h_r = 2\sqrt{1 - x^2}
\]

\[
\frac{k}{b_2 - x} = \frac{y_1 - y_2}{b_1 + b_2} \Rightarrow k = (b_2 - x) \tan \phi
\]

Figure 2.5 The geometry of crack section when \( \phi > 0 \)
The dimensionless local compliance contains only the variables $\bar{x}$ and $\varphi$ so it is defined as:

$$C_{ss}(\bar{x}, \varphi) = \frac{\pi ER C_{ss}}{1 - v^2} = \frac{32}{\pi} \int_0^{\infty} y(1 - x^2) F_z(R) dy dx$$

(2.16)

During the calculation of the compliance of Equation (2.16) it was observed that the results around the zero angles are quite accurate. As the angle of rotation was increased, a divergence of the compliance function value from the expected value was observed. The expected form of the compliance function is known from previous studies [182]. The angle of $\pm 30^\circ$ was then chosen as a limit for our calculation (see Figure 2.6). As it is expected the compliance is calculated when the crack is rotated by $\varphi$ degrees CW or CCW are the same:

$$C_{ss}(\bar{x}, \varphi) = C_{ss}(\bar{x}, -\varphi).$$

On the other hand the geometrical equations mentioned above are valid up to a value of $\varphi$ equal to $\varphi_c$, which is the value where the crack enters in the compression area. This happens when $y_2$ and $y_2 = -b \sin \phi + (R-a) \cos \phi$ and $y_2 = b \cos \phi + (R-a) \sin \phi$. Similarity of triangles in Figure 2.7 gives $k = \frac{y_1 - y_2}{b_2 - b_1} = \frac{(b_2 - b_1) \tan \phi}{b_2 - b_1}$ and $k = (b_2 - b_1) \tan \phi$. The value $C_{ss}$ is calculated up to $\varphi = 30^\circ$ and crack depth $\bar{x} = 0.5$. As the crack rotates more the distance $|b_1 + b_2|$ decreases so the compliance $C_{ss}$ decreases too.
Figure 2.6 The dimensionless compliance $\bar{C}_{55}$ as function of rotational angle for crack depth 0-40%.

\[ h_1 = -b \cos \phi + s, \quad h_2 = b \cos \phi + s \quad s = (R - a) \sin \phi \quad (2.17) \]

\[ y_1 = b \sin \phi + (R - a) \cos \phi, \quad y_2 = b \sin \phi - (R - a) \cos \phi \quad (2.18) \]

The value \( a_s = \frac{h}{2} - k \) remains constant and again the similarity of triangles now gives

\[ \frac{k + y_2}{b_2 - x} = \tan \phi \Rightarrow k = (b_2 - x) \tan \phi - y_2 \] and compliance $\bar{C}_{55}$ is calculated.

<table>
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<th>$\phi_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.04</td>
<td>7.4</td>
</tr>
<tr>
<td>0.08</td>
<td>66.9</td>
</tr>
<tr>
<td>0.12</td>
<td>61.5</td>
</tr>
<tr>
<td>0.16</td>
<td>57.1</td>
</tr>
<tr>
<td>0.20</td>
<td>53.1</td>
</tr>
<tr>
<td>0.24</td>
<td>49.4</td>
</tr>
<tr>
<td>0.28</td>
<td>46.1</td>
</tr>
<tr>
<td>0.32</td>
<td>42.8</td>
</tr>
<tr>
<td>0.36</td>
<td>39.8</td>
</tr>
<tr>
<td>0.40</td>
<td>36.9</td>
</tr>
<tr>
<td>0.44</td>
<td>34.1</td>
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<table>
<thead>
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<th>$\phi_c$</th>
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</thead>
<tbody>
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</tr>
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<td>0.56</td>
<td>26.1</td>
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<td>0.60</td>
<td>23.6</td>
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<td>0.64</td>
<td>21.1</td>
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<td>0.68</td>
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<td>0.72</td>
<td>16.3</td>
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<td>0.76</td>
<td>13.9</td>
</tr>
<tr>
<td>0.80</td>
<td>11.5</td>
</tr>
<tr>
<td>0.84</td>
<td>9.7</td>
</tr>
<tr>
<td>0.88</td>
<td>6.9</td>
</tr>
</tbody>
</table>

Table 2.2: Values of the angle $\phi_c$ vs. the dimensionless crack depth
Chapter 2 – Breathing crack compliance matrix calculation

Figure 2.7 The geometry of crack section when $\phi > \phi_c$.

When the compliance for cracks $a/D > 0.5$ is calculated, a problem appears because of a singularity presented as the variable $x$ approaches the boundaries $b_1$ and $b_2$. In order to avoid this, the variable $\zeta$ where $0 \leq \zeta \leq 1$ is defined and the integration is bounded by $\zeta b_1$ and $\zeta b_2$ [183, 184].

$$\zeta = \frac{\text{Integration Length}}{\text{Crack Length}} \quad (2.19)$$

The compliance $C_{ss}$ is plotted in Figure 2.8 as function of the crack depth for different values of $\zeta$. The value of $\zeta$ recommended is between 90% and 95%.
When $\varphi = 90^\circ$ the compliance value is $C_{55}(90)$ and equal to the compliance value $C_{44}(0)$.

These two situations are exactly the same because when $\varphi = 90^\circ$ the beam is strained in bending by $P_3$ and the local compliance due to the crack is equal to this of bending load $P_3$ and $\varphi = 0$. So $C_{55}(90) = C_{44}(0)$ (see Figure 2.9).

As the crack rotates, the compliance $C_{44}$ is calculated in a respective way of that of $C_{55}$. In Figure 2.10 the crack rotates clockwise and the major part of it lies in the area of positive stretching and remains opened (white part). This has as result the increasing in value of compliance $C_{44}$. 

---

Figure 2.8 The dimensionless compliance $C_{ij}$ as function of crack depth $\alpha$ and $\zeta$. 

![Graph showing dimensionless compliance C as function of crack depth α and ζ.](image)
Chapter 2 – Breathing crack compliance matrix calculation

Figure 2.9 The two equal situations for compliances $\bar{C}_{ss}$ and $\bar{C}_{ss}$. 

Figure 2.10 The geometry of crack section when load is $P_d$ and $\phi > 0$. 

- 95 -
This case is a part of the entire rotation of the crack. The value of \( \overline{C}_{44} \) for a specified \( \phi \) is equal to the value \( \overline{C}_{55} \) for rotational angle \( 90 - \phi \),

\[
\overline{C}_{44}(\phi) = \overline{C}_{55}(90 - \phi)
\]  

(2.20)

The crack depth has no effect to the similarity of these cases. For a constant value of \( \phi \) the integration

\[
\overline{C}_{44}(\overline{a}, \phi) = \frac{\pi E R^3 C_{44}}{1 - \nu^2} = 16 \int_{0}^{\overline{a}} \int_{0}^{\overline{a}} \overline{F}_1^2 \overline{F}_2^2 \overline{F}_2^2 d\overline{x} d\overline{y}
\]

is bounded from \(-\overline{h}\) to \(\overline{h}\) for the variable \(\overline{x}\), and 0 to \(\overline{a}\) for the variable \(\overline{y}\). The only two variables after the integration are \(\overline{a}\) and \(\phi\), so the function \(\overline{C}_{44} = f(\overline{a}, \phi)\) is defined and its values are valid for \(0 < \overline{a} < 1.76\) and \(0 < \phi < 30\). Also the variable \(\zeta\) is necessary in order to avoid the singularity problem. The compliance \(\overline{C}_{44} = f(\overline{a}, \phi)\) is plotted for \(60 < \phi < 90\) with \(0.95 \leq \zeta \leq 1\) (see Figure 2.11).
A new variable $\phi_{cl}$ ($\phi_{\text{closed}}$) must be defined because as the crack rotates further, there is a value for $\phi$ in which the entire crack lies in the area of negative stretch and is considered to be totally closed. There the value of compliance is defined as $\bar{C}_{55} = 0$. The value $\phi_{cl}$ is a function of crack depth $a$ (Table 2.3).

The compliance $\bar{C}_{55}$ is now calculated for three areas of change in the rate of $0^\circ \leq \phi \leq 180^\circ$, which are $0^\circ \leq \phi \leq 30^\circ$, $60^\circ \leq \phi \leq 90^\circ$, $\phi_{cl} \leq \phi \leq 180^\circ$ and are shown in Figure 2.12.

In the diagram in Figure 2.12 there are three points $[0, C_{55}(0)], [\phi_{cl}, C_{55}(\phi_{cl})], [180, 0]$ in which the first derivative of compliance is zero. The natural meaning of zero derivatives in those points are:

$$
\frac{d}{d\phi} \bar{C}_{55} = 0
$$

Figure 2.11 The dimensionless compliance $\bar{C}_{44}$ as function of rotational angle $\phi$ for crack depth 0 to 80% of the radius.
is that the compliance is a value that changes with continuity and smoothness while the crack is rotated. When $\varphi = 0$ or $\varphi = 180^\circ$ the crack passages from the mode of total opening or closing to the mode of semi-closing and semi-opening respectively.

<table>
<thead>
<tr>
<th>$\overline{a}$</th>
<th>$\varphi_{cl}$</th>
<th>$\overline{a}$</th>
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<tr>
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<td>0.08</td>
<td>113.074</td>
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<td>0.12</td>
<td>118.358</td>
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<tr>
<td>0.16</td>
<td>122.860</td>
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<td>0.32</td>
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<td>0.36</td>
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<td>0.40</td>
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<td>145.944</td>
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*Table 2.3. The value of $\varphi_{cl}$ for standard values of crack depth $\overline{a}$*

*Figure 2.12 The dimensionless compliance $C_{ss}$ as function of rotational angle $\varphi$ for crack depth $\overline{a} = 0.3$.**
So the approximation between the areas $30^\circ \leq \varphi \leq 60^\circ$ and $90^\circ \leq \varphi \leq \varphi_i$ follows the boundary conditions of derivative and continuity. The approximation is run using B-Spline curves as shown in Figure 2.13. The compliance $C_{ij}$ is now computable for each value of $\varphi$. After an interpolation of 90 points, the values of the compliance as function of $\varphi$ are shown in following table.

![Figure 2.13 The B-Spline fitting between the defined areas.](image)

The traditional approach is based on a truncated trigonometric fitting of the stiffness of the cracked shaft (Papadopoulos - Dimarogonas, [185]). The known points in this fitting are the stiffness of the shaft with the crack a) closed b) semi-open and c) fully-open as well as the slopes when the crack is closed and open.

$$[K] = [K_0] + [K_1] \cos \omega t + [K_2] \cos 2\omega t + [K_3] \cos 3\omega t + [K_4] \cos 4\omega t$$

where $\omega t = \varphi$ and $[K_0] = \begin{bmatrix} K_{11} & K_{14} & K_{15} \\ K_{41} & K_{44} & K_{45} \\ K_{51} & K_{54} & K_{55} \end{bmatrix}$
Chapter 2 – Breathing crack compliance matrix calculation

<table>
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<tr>
<th>( \phi )</th>
<th>( C_{ss} )</th>
<th>( \phi )</th>
<th>( C_{ss} )</th>
<th>( \phi )</th>
<th>( C_{ss} )</th>
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<td>0.00069</td>
<td>175.95506</td>
<td>0</td>
</tr>
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</table>

Table 2.4 Values of compliance \( C_{ss} \) as function of rotational angle \( \phi \).

The above approach considers that all the stiffness parameters are changing simultaneously (having max and min at the same time), and with the same trigonometric function, according to above relation. The above approach has been used by researches in [186-189]. This is a first approximation to the problem. The crack can be assumed also as a torsional spring that is rotated with the shaft as Gasch did in [190]. Grabowski gave an experimental approach for the compliance change that a crack with depth equal to radius induces. The computed values are presented in Figure 2.14. Some researchers made a general assumption that prescribes the breathing of the crack with two situations, open and close, without any other condition between them. This assumption results in a digital simulation of a rotating shaft with a transverse crack as Chan and Lai presented in [191]. An energy approach of the change of crack compliance while the crack rotates was made by Huang and Shieh in [192]. Also there
is an approach that models the compliance change as cosine, $c_{55}(\phi) = c_{55}(0)(1 + \cos \phi)$, as He, Guo and Chu proposed in [193].

![Figure 2.14 Comparison of Traditional and proposed approach for the change of local compliance](image)

In the present approach the compliance is calculated for every angle of rotation giving thus more accurate results (calculated and no fitted). Each element of the stiffness matrix is changing independently (this means that there are not maximum for every term of the stiffness matrix). Another advantage of current assumption of compliance variation is that the angle of rotation where the crack is closed (or open) is different for every crack angle. In this method the opening and closing of the crack for every depth is clearly specified.

Another way to follow the changing of the stiffness during the rotation is to use a fixed coordinate system. This method is useful for rotors with unequal moment of inertia, or cracked rotors with cracks that remain open during the rotation, but not for the case where the crack opens and closes due to its own weight.
2.2 The cross-coupled compliance of the rotated crack.

A Cartesian coordinate system $XOY$ is considered in Figure 2.15, with point $O$ in the centre of the circular cross section. The angle $\phi$ defines the rotational position of the crack, taking values from 0 to 180 degrees in order to cover all the rotational situations.

The bounds for the integration of Equations (2.16) and (2.21) are now $-\zeta \times b_1$ to $\zeta \times b_2$ for the variable $x$ as in [194], and from $y_2 + k$ to $\sqrt{1-x^2}$ for the variable $y$, defined as follows in Equations (2.22) and (2.23):

\begin{align*}
  b_1 &= b \cos \phi - s, \quad b_2 = b \cos \phi + s, \quad s = (R - a) \sin \phi \\
  y_1 &= b \sin \phi + (R - a) \cos \phi, \quad y_2 = -b \sin \phi + (R - a) \cos \phi
\end{align*} \tag{2.22, 2.23}

Figure 2.15 The geometry of rotated crack section when $\phi > 0$. 

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The local compliance is expressed with respect to coordinate system $X^*O^*Y^*$, which is defined with point $O^*$ being $1-a$ units away from $O$ in the positive $Y$ direction, as in [195, 196]. The dimensionless local compliance contains only the variables $\bar{a}$ and $\varphi$, so it is defined as:

$$
\bar{c}_{45}(\bar{a}, \varphi) = \frac{32}{\pi} \int_{\gamma-\delta}^{\gamma+\delta} \int_{\zeta-\xi}^{\zeta+\xi} \frac{\sqrt{1-\bar{x}^2}}{(\bar{y}-1+\alpha)} \frac{h}{h_1} \frac{F_1}{F_2} \left( \frac{\bar{y}-1+\alpha}{h_1} \right) d\bar{y} 
$$

(2.24)

$$
\bar{c}_{44}(\bar{a}, \varphi) = \frac{16}{\pi} \int_{\gamma-\delta}^{\gamma+\delta} \int_{\zeta-\xi}^{\zeta+\xi} \frac{\sqrt{1-\bar{x}^2}}{(\bar{y}-1+\alpha)} \frac{h}{h_1} \frac{F_1}{F_2} \left( \frac{\bar{y}-1+\alpha}{h_1} \right) d\bar{y} 
$$

(2.25)

$$
\bar{c}_{45}(\bar{a}, \varphi) = \frac{32}{\pi} \int_{0}^{\gamma+\delta} \int_{\zeta+\xi}^{\zeta-\xi} \bar{x} \frac{\sqrt{1-\bar{x}^2}}{(\bar{y}-1+\alpha)} \sqrt{1-\bar{x}^2} \frac{h}{h_1} \frac{F_1}{F_2} \left( \frac{\bar{y}-1+\alpha}{h_1} \right) d\bar{y} 
$$

(2.26)

$$
\bar{c}_{45}(\bar{a}, \varphi) = \frac{32}{\pi} \int_{0}^{\gamma+\delta} \int_{\zeta+\xi}^{\zeta-\xi} \bar{x} \frac{\sqrt{1-\bar{x}^2}}{(\bar{y}-1+\alpha)} \sqrt{1-\bar{x}^2} \frac{h}{h_1} \frac{F_1}{F_2} \left( \frac{\bar{y}-1+\alpha}{h_1} \right) d\bar{y} 
$$

(2.27)

The rotation of the crack is an important issue that strongly affects the coupling phenomenon. Under the steady bending load in the vertical direction, the crack opens and closes as a function of its rotational angle.

Figure 2.16 The breathing of the crack for a) horizontal load and b) vertical load.
When the bending load is horizontal, the opening and closing of the crack occurs in different rotational angles than those for a vertical load. In Figure 2.16, the opening and closing of the crack for both loads is shown as a function of the rotational angle. In these two situations of bending load, there are similar conditions for the crack (opened, closed, semi-open). The value of any compliance for the rotational angles shown in Figure 2.16 for a region encompassing $\phi = \pm 30^\circ$ is computed using Equations (2.24)–(2.27). For the remaining rotational angles, each compliance function is calculated by interpolating the known values using B-splines as in [197,198]. The change of each compliance as a function of the rotational angle and depth of the crack is presented in Figure 2.17. Another way to calculate the compliance function could be by computing the compliance normal to the crack edge and expressing it along the vertical and horizontal direction. This approach is used by many researchers in this field, and results a harmonic periodic compliance change. However, the
compliance change is not exactly harmonic, since has some constant areas during rotation (when the crack is closed) but it is of course periodic. In the present approach the compliance is calculated in each angle of rotation of the shaft giving reliable values.

Importantly, there are rotational angles for which any compliance can equal zero. Compliance \( \tau_{55} \) is equal to zero for the rotational angles that close completely the crack. The same goes for compliance \( \tau_{44} \).

![Figure 2.18 Comparison of three different approaches (Continuous line: A. K. Darpe et al., dashed dot line: Dimarogonas and Papadopoulos, dashed line: current approach) for local compliances \( c_{55}, c_{44} \) and \( c_{45} \) as a function of rotational angle \( \phi \).](image)

Compliance \( \tau_{45} \) takes a value equal to zero not only for a completely closed crack but also for those rotational angles where the crack is symmetric to the plane of the bending load. So, \( \tau_{45}(0) = \tau_{45}(180) = 0 \). In Figure 2.18, the current and traditional approaches for the compliance change during crack rotation are compared with a truncated cosine series in [199] and a recent approach made in [200].
Figure 2.19 Dimensionless Compliances a) $\overline{c}_{44}$, b) $\overline{c}_{45}$, c) $\overline{c}_{54}$ and d) $\overline{c}_{55}$ as a function of rotational angle $\phi$. 
2.3 The assumption about crack form behavior

The current section gives all the detailed information about the different assumption in crack form definition while rotation. As mentioned in the introduction of this chapter the local compliance calculation is exactly the same as presented in chapters 1 and 2 but here there is a bit different assumption in the form of the crack since it is expressed as a function of developed moments and not only as a function of rotational angle as it was presented in previous chapters. In other words, there is an alternative definition about when (time during rotation) the crack obtains full open section.

As shown in Figure 2.20 the rotors elastic line is entirely under the Ox axis (Ox axis passes through the centre of virtual bearings) when the whirling amplitude due to unbalance response is not large enough so as to overtake the static response due to gravity. In this scenario the bending moments acting in the cracked section provoke a static bending due to gravity and a dynamic bending due to unbalance in two planes, the horizontal and the vertical. Note that the bending moments considered in this treatment are three:

The vertical static bending moment due to gravity $M_{Z,G}$

The vertical dynamic bending moment due to unbalance force $M_{Z,U}$

The horizontal dynamic bending moment due to unbalance force $M_{Y,U}$

This assumption about bending moments results in various combinations that can form the crack not only in the form presented in Figure 2.20 but also in others that sometimes can be steady during rotation.

![Figure 2.20 The form of the crack when the elastic line is under the Ox coordinate axis. Note that the centroidal axis is also displaced under the Ox axis.](image)
To explain further, the parameters that define the crack form are two and they are independent to each other:

The crack rotational angle

The resultant bending moment

The first parameter, the crack rotational angle, has nothing to do with what the whirling amplitude or the bending moment values are. In other words, if the motor rotates the rotor crack rotates also in the same angular displacement (matter only of spinning speed). The second parameter, the resultant bending moment, is a function of the three bending moments that mentioned before, but since two of them are of the same plane (vertical), there can be an expression of the resultant bending moment considering two components: the vertical bending moment $M_z$ and the horizontal bending moment $M_y$. The relationship between these two bending moments, $M_y$ and $M_z$, during operation has two scenarios:

1) The vertical bending moment amplitude while rotation is ALWAYS larger than the horizontal bending moment amplitude. This is the case of the inner red ring in Figure 2.21. See also the zoom in Figure 2.22. Such a case can be met when there is large static displacement of the rotor with the whirling amplitude to be small enough due to small unbalance or low speed. In this case the resultant bending moment is ALTERNATIVE and the crack BREATHEs.

*Figure 2.21 The outer and inner “ring” of resultant bending moment developed in the cracked section of the shaft during rotation, together with crack form*
2) The vertical bending moment amplitude while rotation is sometimes larger than the horizontal bending moment amplitude and sometimes smaller. This is the case of the outer red ring in Figure 2.21. Such a case can be met when the whirling amplitude overtakes the static response due to gravity and this can happen when large unbalance is attached in the system, or the rotational speed is higher or during RESONANCE. In such case the resultant bending moment is STATIC with respect to the shaft and the crack form is STEADY.

With the use of Figures 2.21 and 2.22 the phenomenon can be clearly understood under the following stepped consideration:

Having a look in Figure 2.22, the vertical bending moment when no rotation exists lays in axis Mz (see Mz axis in Figure 2.21) and the moment vector has its end in the big red “X” in the centre of the circle.

When rotation starts the unbalance introduces the horizontal dynamic bending moment and the vertical dynamic bending moment, yielding this alternative increment-decrement of My and Mz, yielding the resultant bending moment vector (the green vector) to have its end always around the red circle. Of course the green vector initial point is the black bullet standing for “O” in coordinate system Mz-O-My (this notification is made so as to understand the green vector orientation).

As a consequence, the bending moment vector that acts in the cracked section has an orientation that does not alter much while rotation (see always Figure 2.22).

On the other hand, the crack rotates entirely 360 degrees.

So, by noting the gray shadow in the cracked section (stands for compression area) that is defined from green vector orientation, one can see the alternative intro-outgo of the crack in this area during rotation.

Finally, have a study in the outer ring (See Figure 2.21) and the definition-orientation of the green vector, as long as the gray shadowed area and the crack orientation and …see that the crack has not alternative intro-outgo from the shadowed area of compression but tends ALWAYS to obtain open form.

Keep in mind that the crack initial orientation has to do determinatively with the form of it in the two cases and of course the unbalance orientation has an important role also.
Chapter 2 – Breathing crack compliance matrix calculation

Figure 2.22 A view of the inner "ring" of resultant bending moment developed in the cracked section of the shaft during rotation, together with crack form

Under the current consideration the relative angle of the crack with respect to bending moment vector (green vector) is now defined as “compliance angle” $\varphi_{\text{com}}$ and expresses exactly the same property as the rotational angle in previous chapters (when the bending moment vector was assumed steady). With respect to the angle $\varphi_{\text{com}}$ the local compliance matrix is defined from the “data base” already existing from the previous chapters and the compliance values are defined for every rotational step. These compliance values express the compliance values with respect to the bending moment plane (green vector) and not with respect to planes Y and Z as it is demanded. So a last step is to express the compliance values with respect to planes Y and Z with a usual transformation as follows.

There are several angular magnitudes that have to be defined so as the current crack behavior to be modeled. Considering also Figure 2.23 the following angles can be defined with respect to bending moment values that coincide four different cases according to the quadrant the moment vector lays to. The bending moment angle $\varphi_m$ is defined as follows for the four different cases. Also the angle $\varphi_m'$ is defined further.
If \( M_y > 0 \) and \( M_z > 0 \) then \( \varphi_m = \arctan\left(\frac{|M_y|}{|M_z|}\right) \cdot 180 / \pi, \quad \varphi'_m = \varphi_m + 180 \)

If \( M_y > 0 \) and \( M_z < 0 \) then \( \varphi_m = 180 - \arctan\left(\frac{|M_y|}{|M_z|}\right) \cdot 180 / \pi, \quad \varphi'_m = \varphi_m + 180 \)

If \( M_y < 0 \) and \( M_z < 0 \) then \( \varphi_m = \arctan\left(\frac{|M_y|}{|M_z|}\right) \cdot 180 / \pi + 180, \quad \varphi'_m = \varphi_m - 180 \)

If \( M_y < 0 \) and \( M_z > 0 \) then \( \varphi_m = 360 - \arctan\left(\frac{|M_y|}{|M_z|}\right) \cdot 180 / \pi, \quad \varphi'_m = \varphi_m - 180 \)

According to the rotational angle of the crack \( \psi \) (as it was defined in previous chapters) the new rotational angle \( \varphi_c \) is defined as follows:

If \( 0 \leq \psi \leq 90 \) then \( \varphi_c = \psi + 90 \)

If \( 90 < \psi \leq 180 \) then \( \varphi_c = \psi + 90 \)

If \( 180 < \psi \leq 270 \) then \( \varphi_c = \psi + 90 \)

If \( 270 < \psi < 360 \) then \( \varphi_c = \psi - 270 \)

According to the respective value of \( \varphi'_m \) to \( \psi \) the angle \( \varphi_{com} \) that coincides with the compliance value is defined as:

If \( \psi < \varphi'_m \) then \( \varphi_{com} = \varphi'_m - \psi + 180 \)

If \( \psi > \varphi'_m \) then \( \varphi_{com} = 180 - \psi + \varphi'_m \)
Chapter 2 – Breathing crack compliance matrix calculation

Under the previous definition progress the compliance values are defined according to the angle \( \varphi_{\text{COM}} \) and then they have to be transformed in the vertical and horizontal plane as follows:

\[
\begin{align*}
\begin{bmatrix}
    c_{xs}(\varphi_{\text{COM}}) \\
    c_{ys}(\varphi_{\text{COM}})
\end{bmatrix} &=
\begin{bmatrix}
    \sin(\varphi_s) & \cos(\varphi_s) \\
    -\cos(\varphi_s) & \sin(\varphi_s)
\end{bmatrix}
\begin{bmatrix}
    c_{s1}(\varphi_{\text{COM}}) \\
    c_{s2}(\varphi_{\text{COM}})
\end{bmatrix}, \\
\begin{bmatrix}
    c_{zs}(\varphi_{\text{COM}}) \\
    c_{zs}(\varphi_{\text{COM}})
\end{bmatrix} &=
\begin{bmatrix}
    \sin(\varphi_s) & \cos(\varphi_s) \\
    -\cos(\varphi_s) & \sin(\varphi_s)
\end{bmatrix}
\begin{bmatrix}
    c_{s4}(\varphi_{\text{COM}}) \\
    c_{s4}(\varphi_{\text{COM}})
\end{bmatrix},
\end{align*}
\]

In order to have an overview of how the present crack form simulation functions a simple application with a hinged-hinged model of shaft of chapter 1 is presented.

Consider the hinged-hinged shaft carrying a disc, as in Figure 2.20. A start up is made and the time histories of disc centre response are plotted in Figure 2.24 for vertical and horizontal plane.

Next, in Figure 2.24 the bending moments developed in the cracked section in vertical and horizontal plane are plotted as function of time. As it is seen from Figures 2.23 and 2.24 the displacement in vertical plane gets near to zero as the system gets near to resonance of first critical speed. In this time period, before and after resonance the corresponding bending moment in vertical plane becomes with negative values also that means enters the “outer ring” of Figure 2.21. It is worthwhile to see the progress of the angles demanded for compliance definition. In Figure 2.25 the value of \( \varphi_s \) and of \( \varphi_m \) is plotted for the entire time domain with two time periods of interest to be highlighted, this of before resonance and this of during resonance.

As it is shown in Figure 2.25 the amplitude of the angle progress is limited before resonance in about 180 degrees with a small variation (inner ring) but as the system gets to resonance the variation obtains the entire domain of 0 up to 360 degrees (outer ring). Similar progress is noticed also in Figure 2.26 about the angle \( \varphi_m \) . Calculating and plotting the compliance angle \( \varphi_{\text{COM}} \) in Figure 2.26 and the corresponding compliance values in Figure 2.29 the pass from breathing behavior to steady open form is presented. The fact that the crack obtains the open form (compliances are steadily near maximum) has to do with the initial crack orientation. If the crack orientation is different then the steady form through resonance would be different (closed, semi closed, etc).
In this manner the crack behavior is simulated better since in real systems is not always breathing. As mentioned before, the experimental procedure with crack does not involves breathing crack but a steadily open cut, so the current simulation offers the ability of steadily open crack that yields similar treatment about coupling phenomenon as explained in present chapter.

**Figure 2.24** a) Vertical and b) Horizontal response time history for disc center. Note the red line of zero absolute displacement in (a).

**Figure 2.25** a) Vertical and b) Horizontal response time history for bending moment. Note the red line of zero absolute bending moment in (a).
Figure 2.26 $\phi_{M}'$ values as a function of time. Note that the domain of progress changes covering the entire 360 degrees as the system gets to resonance.
Figure 2.27 $\phi_{M}$ values as a function of time. Note that the domain of progress changes covering the entire 360 degrees as the system gets to resonance.
Figure 2.28 $\varphi_{\text{COM}}$ values as a function of time. Note that the domain of progress changes covering values around 120 degrees as the system gets to resonance. This progress (in the right) yields steady crack form while the other (in the left) yields breathing.
Figure 2.29 Compliance $c_{55}$ values as a function of time. Note that the domain of progress changes from entire value domain (breathing) to limited variation for steadily open crack. Similar progress is noticed for the other three compliances also.
Chapter 3

Coupling Effects in Cracked Beams and Simple Rotor-Bearing Systems

In the present chapter the dynamic behavior of a cracked Euler-Bernoulli beam with two transverse surface cracks is studied initially in order to investigate coupling phenomenon of bending vibrations in two main planes. Each crack is characterized by its depth, position and relative angle. Both cracks are considered to lie in arbitrary angular positions with respect to the longitudinal axis of the beam and at any distance from the left clamped end. The main target is to inspect the role of crack rotational angle in coupling presence and to investigate the shift of natural frequencies and frequency response during the presence of coupling due to rotational angle.

An experimental procedure of a clamped-free beam of circular section is presented also in this chapter so as to present the coupling phenomenon in a real situation since the role of coupling due to crack is a significant matter in this dissertation considering that the coupling became at last the tool for crack detection in rotor bearing systems.

Additionally, simple rotor bearing systems with a crack are investigated towards critical speed estimation and frequency response shift in order to achieve an inspection of coupling phenomenon effects in continuous (Rayleigh) rotor vibrations mounted in linear bearings. In contrast to the model of rotor bearing system extensively presented to chapter 1, in this chapter linear bearing consideration is assumed so as to compose a more simple system for the reason that linear bearings offer specific and known (predefined) stiffness and damping properties. The bearing properties can produce coupled vibrations since coupled bearing coefficients are introduced in the bearing model. The separation of the coupling due to crack and coupling due to bearings is made using the frequency response of the coupled system as it will be shown in detail.
3.1 Cracked Beam Bending Vibration

Consider a beam with two cracks at distance $L_1$ and $L_2$ respectively from the clamped end. The cracks divide the beam into three parts with bending displacements $Y_i(x,t)$, $i = 1, 2, 3$. Each part is connected with the next by springs $K_i$ with magnitude depending on the crack depth and the crack angular position (see Figure 3.1).

![Figure 3.1 The Euler-Bernoulli beam with two cracks.](image)

The three parts of the beam vibrate with $Y_1(x,t)$, $Y_2(x,t)$, $Y_3(x,t)$ respectively. The bending moment $P_5$ is applied (the bending moment is inserted in boundary condition as it will be shown later) and the vibration is described by the well known equation of Euler-Bernoulli:

$$\frac{\partial^4 Y_i}{\partial x^4} = \frac{1}{C_y} \frac{\partial^2 Y_i}{\partial t^2} \quad \text{for} \ i = 1, \ 0 < x < L_1$$

$$\quad \text{where} \quad \text{for} \ i = 2, \ L_1 < x < L_2$$

$$\quad \text{for} \ i = 3, \ L_2 < x < L$$

(3.1)

Where $i = 1, 2, 3$ is the part of the beam, $C_y = \sqrt{EI/\mu}$, $\mu = A\rho$, $A = \pi R^2$, $I = \pi R^4/4$, $E = 210\, GPa$ is the Young modulus of Elasticity, $I$ is the moment of inertia of the cross section of the beam, $\mu$ is the linear density, the density of the material is $\rho = 7800\, Kg/m^3$ (steel), $A$ is
Chapter 3 – Coupling effects in cracked beams and simple rotor-bearing systems

the cross section of the shaft, and \( R \) is the shaft radius. Defining as \( \tilde{x} = x / L \) and \( \tilde{y} = y / L \) the dimensionless differential Euler-Bernoulli equations are obtained:

\[
\frac{\partial^4 \tilde{y}}{\partial \tilde{x}^4} = \frac{1}{C_y} \frac{\partial^2 \tilde{y}}{\partial \tau^2} \quad \text{for } i = 1, \ 0 < \tilde{x} < b_1
\]

\[
\frac{\partial^4 \tilde{y}}{\partial \tilde{x}^4} = \frac{1}{C_y} \frac{\partial^2 \tilde{y}}{\partial \tau^2} \quad \text{for } i = 2, \ b_1 < \tilde{x} < b_2
\]

\[
\frac{\partial^4 \tilde{y}}{\partial \tilde{x}^4} = \frac{1}{C_y} \frac{\partial^2 \tilde{y}}{\partial \tau^2} \quad \text{for } i = 3, \ b_2 < \tilde{x} < 1
\]

(3.2)

Where \( C_y = C_{y,T} / L^2 \) and \( \tau = \frac{t}{T} \).

3.1.1 Boundary conditions and characteristic determinant

After the separation of the variables the three partial solutions of each one part are given by,

\[
\tilde{Y}_1(\tilde{x}) = A_1 \cosh(\tilde{k}_1 \tilde{x}) + A_2 \sinh(\tilde{k}_1 \tilde{x}) + A_3 \cos(\tilde{k}_1 \tilde{x}) + A_4 \sin(\tilde{k}_1 \tilde{x}),
\]

(3.3)

\[
\tilde{Y}_2(\tilde{x}) = A_1 \cosh(\tilde{k}_2 \tilde{x}) + A_2 \sinh(\tilde{k}_2 \tilde{x}) + A_3 \cos(\tilde{k}_2 \tilde{x}) + A_4 \sin(\tilde{k}_2 \tilde{x}),
\]

(3.4)

\[
\tilde{Y}_3(\tilde{x}) = A_1 \cosh(\tilde{k}_3 \tilde{x}) + A_2 \sinh(\tilde{k}_3 \tilde{x}) + A_3 \cos(\tilde{k}_3 \tilde{x}) + A_4 \sin(\tilde{k}_3 \tilde{x}),
\]

(3.5)

Where \( \tilde{k}_i = 2\pi / C_y, \ \tilde{Y}_1 = Y_1 / L, \ \tilde{Y}_2 = Y_2 / L, \ \tilde{Y}_3 = Y_3 / L, \ b_1 = L_1 / L \) and \( b_2 = L_2 / L \).

At the clamped end the displacement and the slope are zero, and at the free end the moment and the tangent force are also zero. At the positions of the two cracks the continuity of the displacements, the slopes, the moments and the tangent forces must be satisfied. The boundary conditions are also given in dimensionless form,

\[
\tilde{Y}_1(0) = 0 \quad (3.6) \quad \tilde{Y}_1^*(b_1) = \tilde{Y}_2^*(b_1)
\]

\[
\tilde{Y}_2(0) = 0 \quad (3.7) \quad \tilde{Y}_2^*(b_2) = \tilde{Y}_1^*(b_2)
\]

\[
\tilde{Y}_3(1) = 0 \quad (3.8) \quad \tilde{Y}_3^*(b_2) = \tilde{Y}_1^*(b_2)
\]

\[
\tilde{Y}_1^*(1) = 0 \quad (3.9) \quad \tilde{Y}_1^*(b_1) = \tilde{Y}_3^*(b_1)
\]

\[
\tilde{Y}_2^*(1) = 0 \quad (3.10) \quad \tilde{Y}_2^*(b_2) = \tilde{Y}_3^*(b_2)
\]

\[
\tilde{M}_1 = \frac{R(1-v^2)}{4L} C_{S5}, \quad i = 1,2 \text{ with } \tilde{C}_{S5} \text{ the dimensionless compliance of the first (} i = 1) \text{ and the second crack (} i = 2) \text{ and } v = 0.3 \text{ is the Poisson ratio for steel. Combining the Equations (3.3), (3.4) and (3.5) with the twelve boundary conditions, twelve homogeneous equations are obtained, with the corresponding unknown coefficients } A_1, A_2, \ldots, A_4. \text{ In order to}
obtain non trivial solution for the system the characteristic equation must be equal to zero. Each line of the characteristic equation corresponds to one boundary condition and each column corresponds to one unknown coefficient \( A_i, i = 1, 2, \ldots, 12 \).

\[
\begin{bmatrix}
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

where \( c() = \cos(), s() = \sin(), ch() = \cosh(), sh() = \sinh() \).

The terms \( S_{ij} \) in the determinant are given by the equations:

\[
(3.19) \quad S_{8,1} = M_{55} \bar{k}_y \text{ch}(\bar{k}_y b_1) + \text{sh}(\bar{k}_y b_1) \\
(3.20) \quad S_{12,5} = M_{55} \bar{k}_y \text{ch}(\bar{k}_y b_2) + \text{sh}(\bar{k}_y b_2) \\
(3.21) \quad S_{8,2} = M_{55} \bar{k}_y \text{sh}(\bar{k}_y b_1) + \text{ch}(\bar{k}_y b_1) \\
(3.22) \quad S_{12,6} = M_{55} \bar{k}_y \text{sh}(\bar{k}_y b_2) + \text{ch}(\bar{k}_y b_2)
\]

The roots of the determinant are the dimensionless eigenvalues of the multi-cracked shaft.

Since \( \bar{k}_1^2 = \frac{2\pi}{C_y}, \bar{k}_2^2 = \frac{2\pi}{C_y T / L'}, \omega = \frac{\omega_0}{\omega_0} \), where \( \omega_0 = \sqrt{EI / \mu L'} \). Thus the calculation of the eigenvalues, as functions of the crack parameters is feasible.

### 3.1.2 Calculation of the eigenfrequencies

Consider the relative positions of the first and the second crack as \( b_1 = 0.1 \) and \( b_2 = 0.4 \) respectively, both relative crack depths equal to \( \bar{a}_1 = \bar{a}_2 = 0.3 \) and the rotational angle of both cracks equal to \( \varphi_1 = \varphi_2 = 30^\circ \). Then the dimensionless compliance for both cracks is \( \bar{C}_{55} = 0.34633 \) and the dimensionless flexural compliance is \( M_{55} = 0.000964421 \).

The characteristic determinant is calculated and plotted in Figure 17. The roots of equation \( \text{Det}[Q] = 0 \) are calculated using the Newton-Raphson method and the first, second and third
are found equal to \( \Omega_1 = \frac{\omega_1}{\omega_n} = 1.87388 \), \( \Omega_2 = \frac{\omega_2}{\omega_n} = 4.69128 \) and \( \Omega_3 = \frac{\omega_3}{\omega_n} = 7.85267 \) which are the first, second, and third natural frequencies of the vibrating cracked beam. In the case of an uncracked beam \((h_1 = h_2 = 1, \bar{C}_{ss_1} = \bar{C}_{ss_2} = 0)\) the characteristic determinant of the system is

\[
\text{Det}(Q) = 32 \left[ \cosh(k_y) \cdot \cos(k_y) + 1 \right]
\]

and the roots are given approximately by \( k_y = (i - 1/2) \cdot \pi \), where \( i = 1, 2, ..., n \). The approximation is as higher as high \( i \) is. In Table 3.1 the first six dimensionless eigenfrequencies are shown as result from the calculation of the determinant.

<table>
<thead>
<tr>
<th>( \bar{k}_{y1} )</th>
<th>( \bar{k}_{y2} )</th>
<th>( \bar{k}_{y3} )</th>
<th>( \bar{k}_{y4} )</th>
<th>( \bar{k}_{y5} )</th>
<th>( \bar{k}_{y6} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.851</td>
<td>4.694</td>
<td>7.855</td>
<td>10.996</td>
<td>14.137</td>
<td>17.279</td>
</tr>
</tbody>
</table>

*Table 3.1 Values of the first six dimensionless eigenfrequencies of an uncracked beam.*

![Graph of characteristic determinant](image)

**Figure 3.2** The characteristic determinant of a beam with two cracks in relative positions \( b_1 = 0.1 \) and \( b_2 = 0.4 \), relative depths \( \bar{a}_1 = \bar{a}_2 = 0.3 \), and rotational angles \( \phi_1 = \phi_2 = 30^\circ \).
It is expected that the natural eigenfrequency of a cracked beam are lower than those of the uncracked one. This is a fact due to the reduction of beam’s stiffness caused by the crack existence. This is visible in (Figure 3.2) where the natural frequencies are a bit lower than those in Table 3.1. Also, regarding that natural frequency is a function of stiffness, a deeper crack causes greater reduction in natural frequency.

There are six variables that describe clearly the existence of two transverse cracks in a beam. These are the two depths $a_1, a_2$, the two longitudinal positions $b_1, b_2$ and the two rotational angles $\phi_1, \phi_2$. In following, the way each of these six variables affect the vibration will be investigated. Considering that both cracks are rotated independently in the space $0 \leq \phi \leq 180$ and the crack depths are equal $a_1 = a_2 = 0.3$, the $1^{st}$ eigenfrequency can be calculated with respect to the change of rotating position. Calculating the eigenfrequency for a step change of 10 degrees for each cracks rotational angle, 56 points are obtained. After a $7^{th}$ order interpolating surface, Figure 3.3 is obtained. It is supposed that the cracks longitudinal positions are constant as in previous numerical examples.

![Figure 3.3](image)

**Figure 3.3** First dimensionless eigenfrequency as function of each crack’s rotational angle $\phi_1$ and $\phi_2$, with crack depths $a_1 = a_2 = 0.3$ and crack position $b_1 = 0.1$ and $b_2 = 0.4$

The same procedure can obviously be repeated for all the positions, depths, and rotational angles of the cracks. As shown in Figure 3.3 the change of First Natural Eigenfrequency (FNE) is stronger due to the first crack and weaker due to the second one. The nearer to the bounded end the crack appears, the greater decrement the (FNE) has. This fact is due to the
internal flexural torsion that is developed while bending, a measure that is greater near the clamped end. If both cracks are moved towards the free end, let’s say $b_1 = 0.25$ and $b_2 = 0.75$ by keeping the rest parameters constant, the respective diagram in Figure 3.4 verifies that the change of (FNE) due to the faraway from the clamped end crack is negligible.

Another fact is that when both cracks tend to close ($\phi_1 = \phi_2 \approx 180^\circ$), the (FNE) gets the value of $\Omega_1 = 1.87516 = \Omega_{(uncracked)}$ that is obvious as long as the local compliance of closed crack is considered zero. In order to get a view of how strongly the position of the crack affects the eigenfrequency change, Figure 3.5 is designed.

In Figure 3.6 it is shown that the crack position affects more the natural eigenfrequencies than the crack depth does. There is a change of the order of 0.01 when the cracks are moved on the beam, whereas the crack depth alteration changes the eigenfrequencies in an amount of 0.001.

In next, three of the six crack parameters vary in order to observe the affect to (FNE). One crack is kept constant ($a_1 = 0.1$, $b_1 = 0.05$, $\phi_1 = 0^\circ$) and the characteristics of the other vary ($0 \leq a_2 \leq 0.4$, $0.05 \leq b_2 \leq 1$, $0 \leq \phi_2 \leq 180^\circ$). Each diagram in Figure 21(a-h) is a result of constant crack position $b_2$.

**Figure 3.4** First dimensionless eigenfrequency as function of each crack’s rotational angle $\phi_1$ and $\phi_2$, with crack depths $a_1 = a_2 = 0.3$ and crack position $b_1 = 0.25$ and $b_2 = 0.75$.

**Figure 3.5** 1st dimensionless eigenfrequency as function of each crack’s position $b_1$ and $b_2$, with crack depths $a_1 = a_2 = 0.3$ and crack rotational angles $\phi_1 = \phi_2 = 0$. 

- 125 -
In Figure 3.6(a-h) it is observed that the lower value of (FNE) (point $\phi = 0^\circ, \alpha_2 = 0.4$) increases as long as the crack approaches the free end of the beam. From the position $b_2 = 0.65$ and on, the influence of second crack in (FNE) is negligible for the crack depth $\alpha_2 = 0.4$. The diagram is almost flat and the eigenfrequency is affected only by the first crack.
3.1.3 Determination of the eigenmodes

After the determination of the natural frequencies of a cracked beam the eigenmodes of vibration can be calculated. The eigenmodes are the tool for diagnosing the crack position on the beam. This happens given that the eigenmodes present a slope discontinuity at the points where the crack exists. This discontinuity is stronger as the crack depth increases and the crack is open, which means that the local compliance takes the greater value from every other situation of rotation. The eigenmodes of a cracked beam with two cracks are given by the Equations (3.3), (3.4) and (3.5) in which the value of the natural frequency is substituted \(\Omega_i \). The values of \(A_1, A_2, ..., A_{12}\) are determined by solving the linear system of twelve boundary conditions (3.6), (3.7) ... (3.17). The homogenous system is defined as following.

\[
[Q]\{A_1\, A_2\, A_3\, A_4\, A_5\, A_6\, A_7\, A_8\, A_9\, A_{10}\, A_{11}\, A_{12}\} = 0
\]

(3.27)

Because of the fact that \(\det[Q] = 0\) when \(\bar{K}_n = \sqrt{\Omega_i} \), the matrix \([Q]\) is non invertible and the solution is achieved by setting \(A_{12} = 1\) for each eigenfrequency \(\Omega_j\), \(j = 1, 2, ..., N\). Then, the system equations are eleven and the variables \(A_{1j}^{(j)}\) are moved on to the second part of equations. The solution of \(A_{1j}^{(j)}, A_{2j}^{(j)}, ..., A_{12j}^{(j)}\) is now feasible. In the following numerical example two cracks with \(a_1 = a_2 = 0.5\,\text{, } b_1 = 0.1\,\text{, } b_2 = 0.4\,\text{, } \phi_1 = \phi_2 = 0^\circ\) are considered. The first three natural frequencies are determined equal to \(\Omega_1 = 1.82545\,\text{, } \Omega_2 = 4.61143\,\text{, } \Omega_3 = 7.79549\).
Equations (3.3), (3.4) and (3.5) give the first eigenmode for $K_y = \sqrt{\Omega_1}$, the second eigenmode for $K_y = \sqrt{\Omega_2}$, and the third eigenmode for $K_y = \sqrt{\Omega_3}$ (see Figure 3.7 (a-c)).

The slope discontinuity is not clearly visible in the two crack positions $b_1 = 0.1$, $b_2 = 0.4$ because of the small crack depth. In these positions a discontinuity appears on the eigenmode and becomes sharper as the crack depth increases. The tracing of this slope discontinuity is feasible using wavelets transformation.
3.2 Coupled Bending Vibrations of a Stationary shaft

In this section the coupled bending vibrations of a stationary shaft with two cracks is studied. It is known from the literature that, when a crack exists in a shaft, the bending, torsional, and longitudinal vibrations are coupled. This study focuses on the horizontal and vertical planes of a cracked shaft, whose bending vibrations are caused by a vertical excitation, in the clamped end of the model. When the crack orientations are not symmetrical to the vertical plane, a response in the horizontal plane is observed due to the presence of the cracks. The crack orientation is defined by the rotational angle of the crack, a parameter which affects the horizontal response. When more cracks appear in a shaft, then the coupling becomes stronger or weaker depending on the relative crack orientations. It is shown that a double peak appears in the vibration spectrum of a cracked or multi-cracked shaft. Modeling the crack in the traditional manner, as a spring, yields analytical results for the horizontal response as a function of the rotational angle and the depths of the two cracks. A 2X2 compliance matrix, containing two non-diagonal terms (those responsible for the coupling) serves to model the crack. Using the Euler–Bernoulli beam theory, the equations for the natural frequencies and the coupled response of the shaft are defined. The experimental coupled response and eigenfrequency measurements for the corresponding planes are presented. The double peak was also experimentally observed.

Consider a clamped-free stationary shaft with two transverse cracks at distance \( L_1 \) and \( L_2 \), respectively, from the clamped end. A Cartesian coordinate system is defined as in Figure 3.22; the vertical plane \( Oxy \) is defined as plane 5 and horizontal plane \( Oxz \) is defined as plane 4. The bending vibrations occur in both planes under excitation in plane 5. The cracks divide the shaft into three parts with vertical displacements \( Y_i(x,t), i=1,2,3 \) and horizontal displacements \( Z_i(x,t), i=1,2,3 \) as shown in Figure 3.22. Each part is connected with the next by spring \( K_i \), whose magnitude depends on the crack depth and the crack angular position as in Figure 3.22. The magnitude of \( K_i \) is different for the vertical and horizontal plane. The three parts of the shaft vibrate in the vertical plane (plane 5) with \( Y_1(x,t), Y_2(x,t), \) and \( Y_3(x,t) \), and in the horizontal plane (plane 4) with \( Z_1(x,t), Z_2(x,t), \) and \( Z_3(x,t) \). When there isn’t crack, the shaft is considered to vibrate independently in two planes. In this case the response exists exclusively in the plane of excitation. So, the Equations of motion as shown below are not coupled as happens in rotating shafts.
The bending moment $P_5$ is applied at the free end. The vibration is described by the Euler-Bernoulli Equations for the vertical and horizontal plane in Equation (3.30).

$$\frac{\partial^4 Y_i}{\partial x^4} = \frac{1}{C_i^2} \frac{\partial^2 Y_i}{\partial t^2}, \quad -\frac{\partial^4 Z_i}{\partial x^4} = \frac{1}{C_i^2} \frac{\partial^2 Z_i}{\partial t^2}, \quad \text{where} \quad \begin{array}{ll}
\text{for } i = 1, & 0 \leq x \leq L_1 \\
\text{for } i = 2, & L_1 < x \leq L_2 \\
\text{for } i = 3, & L_2 < x \leq L
\end{array}$$  \hspace{1cm} (3.30)

Where $i = 1, 2, 3$ is the part of the beam, $C_i = C_z = \sqrt{EI/\mu}$, $\mu = A\rho$, $A = \pi R^2$, $I = \pi R^4/4$, $E = 210$ GPa is the Young’s modulus of elasticity, $I$ is the moment of inertia of the entire cross section area of the shaft, $\rho$ is the linear density, the density of the material is $\rho = 7860$ kg.m$^{-3}$, $A$ is the entire cross section area of the shaft, and $R$ is the shaft radius.

After the separation of variables, the three partial solutions for each part of the vertical plane are:

$$Y_1(x) = A_1 \cosh(k_1 x) + A_2 \sinh(k_1 x) + A_3 \cos(k_1 x) + A_4 \sin(k_1 x)$$  \hspace{1cm} (3.31)

$$Y_2(x) = A_5 \cosh(k_2 x) + A_6 \sinh(k_2 x) + A_7 \cos(k_2 x) + A_8 \sin(k_2 x)$$  \hspace{1cm} (3.32)

$$Y_3(x) = A_9 \cosh(k_3 x) + A_{10} \sinh(k_3 x) + A_{11} \cos(k_3 x) + A_{12} \sin(k_3 x)$$  \hspace{1cm} (3.33)

The three partial solutions for each part of the horizontal plane are:

$$Z_1(x) = B_1 \cosh(k_1 x) + B_2 \sinh(k_1 x) + B_3 \cos(k_1 x) + B_4 \sin(k_1 x)$$  \hspace{1cm} (3.34)

$$Z_2(x) = B_5 \cosh(k_2 x) + B_6 \sinh(k_2 x) + B_7 \cos(k_2 x) + B_8 \sin(k_2 x)$$  \hspace{1cm} (3.35)
Chapter 3 – Coupling effects in cracked beams and simple rotor-bearing systems

\[ Z_z(x) = B_0 \cosh(k_z x) + B_1 \sinh(k_z x) + B_2 \cos(k_z x) + B_3 \sin(k_z x) \]  

(3.36)

where \( k_z = \sqrt{\frac{\omega}{C_v}} \), \( k_z = \sqrt{\frac{\omega}{C_s}} \) and \( \omega \) is the vibration frequency.

3.2.1 Boundary conditions and the characteristic determinant.

There are 24 unknown variables in the above Equations and the solution for the Equations of motion can be calculated using 24 boundary conditions, 12 for each plane (see Table 3.2). The boundary conditions are similar for both planes. In equations of Table 3.2, \( c_{ij0} \) and \( c_{ij0} \ (i = 4, 5) \) are the local compliances of the first and second crack correspondingly and they are defined as in Equation (3.37).

\[ c_{ij} = \frac{(1 - \nu^2)}{ER} c_{ij0} \]  

(3.37)

<table>
<thead>
<tr>
<th>Vertical Boundary Conditions</th>
<th>Horizontal Boundary Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y_1(0) = 0 )</td>
<td>( Z_1(0) = 0 )</td>
</tr>
<tr>
<td>( Y_1(0) = 0 )</td>
<td>( Z_1'(0) = 0 )</td>
</tr>
<tr>
<td>( Y_1'(L) = 0 )</td>
<td>( Z_1'(L) = 0 )</td>
</tr>
<tr>
<td>( Y_1''(L) = 0 )</td>
<td>( Z_1''(L) = 0 )</td>
</tr>
<tr>
<td>( Y_2(L) = Y_2(L_1) )</td>
<td>( Z_2(L) = Z_2(L_1) )</td>
</tr>
<tr>
<td>( Y_2'(L) = Y_2'(L_1) )</td>
<td>( Z_2'(L) = Z_2'(L_1) )</td>
</tr>
<tr>
<td>( Y_2''(L) = Y_2''(L_1) )</td>
<td>( Z_2''(L) = Z_2''(L_1) )</td>
</tr>
<tr>
<td>( E_l c_{55} Y_1''(L_1) + E_l c_{54} Z_1''(L_1) = \Delta Y_1'(L_1) )</td>
<td>( E_l c_{44} Z_1''(L_1) + E_l c_{43} Y_1''(L_1) = \Delta Z_1'(L_1) )</td>
</tr>
<tr>
<td>( Y_2(L_2) = Y_2(L_2) )</td>
<td>( Z_2(L_2) = Z_2(L_2) )</td>
</tr>
<tr>
<td>( Y_2'(L_2) = Y_2'(L_2) )</td>
<td>( Z_2'(L_2) = Z_2'(L_2) )</td>
</tr>
<tr>
<td>( Y_2''(L_2) = Y_2''(L_2))</td>
<td>( Z_2''(L_2) = Z_2''(L_2) )</td>
</tr>
<tr>
<td>( E_l c_{55} Y_2''(L_2) + E_l c_{54} Z_2''(L_2) = \Delta Y_2'(L_2) )</td>
<td>( E_l c_{44} Z_2''(L_2) + E_l c_{43} Y_2''(L_2) = \Delta Z_2'(L_2) )</td>
</tr>
</tbody>
</table>

Table 3.2 Boundary conditions for a clamped-free shaft with a crack in the two main directions of vibration
Equations (3.31) through (3.36) are substituted into the 24 boundary conditions of Table 3.2 to obtain the homogenous system of 24 equations and 24 unknowns \((A_1, A_2, ..., A_{12}, B_1, B_2, ..., B_{12})\) in Equation (3.38).

\[
\begin{bmatrix}
A_1 & A_2 & \cdots & A_{12} & B_1 & B_2 & \cdots & B_{12}
\end{bmatrix}^T \cdot
\begin{bmatrix}
0 & 0 & \cdots & 0
\end{bmatrix}^T = \mathbf{P}
\]  
\[(3.38)\]

In Table 3.2, \(\Delta Y_j = Y_j - Y_i\), \(\Delta Z_j = Z_j - Z_i\) where \(i = 1, 2, j = 1, 2,\).

The characteristic determinant of the homogenous system in Equation (3.39), \(\det(\mathbf{P})\), must be equal to zero for a non-trivial solution. The roots of the characteristic equation \(\det(\mathbf{P}) = 0\) are the eigenfrequencies of the multi-cracked shaft. Matrix \(\mathbf{P}\) in (3.39) is defined in Appendix B.

\[
\mathbf{P}_{24 \times 24} = \begin{bmatrix}
A_{12 \times 12} & B_{12 \times 12} \\
C_{12 \times 12} & D_{12 \times 12}
\end{bmatrix}
\]  
\[(3.39)\]

### 3.2.2 Calculation of the eigenfrequencies of the multicracked shaft.

Considering a shaft with two cracks of any characteristics (position, depth, rotational angle) the dimensionless compliances \(\bar{c}_{55}, \bar{c}_{44}, \bar{c}_{54}, \bar{c}_{45}\) can be computed for both cracks and the characteristic determinant becomes a function of frequency \(\omega\). There are areas of rotational angle in which the dimensionless compliances are equal to zero. The physical explanation of zero compliance is that the crack does not introduce any additional slope in the shaft in the specific direction, and so the term of the additional slope due to coupling in the boundary conditions of Table 3.2 becomes equal to zero. The same goes for the other terms that contain the dimensionless compliances \(\bar{c}_{55}\) and \(\bar{c}_{44}\).

<table>
<thead>
<tr>
<th>Crack depth (\alpha_i / D = \alpha_j / D)</th>
<th>1(^{st}) eigenfrequency Vertical / Horizontal</th>
<th>2(^{nd}) eigenfrequency Vertical / Horizontal</th>
<th>3(^{rd}) eigenfrequency Vertical / Horizontal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>226.09 / 226.09</td>
<td>1416.88 / 1416.88</td>
<td>3967.31 / 3967.31</td>
</tr>
<tr>
<td>0.1</td>
<td>225.65 / 226.05</td>
<td>1415.1 / 1416.8</td>
<td>3965.1 / 3967.15</td>
</tr>
<tr>
<td>0.2</td>
<td>223.9 / 225.76</td>
<td>1407.9 / 1415.56</td>
<td>3956.2 / 3965.65</td>
</tr>
<tr>
<td>0.25</td>
<td>222.4 / 225.35</td>
<td>1401.75 / 1413.87</td>
<td>3948.63 / 3963.6</td>
</tr>
<tr>
<td>0.3</td>
<td>220.4 / 224.6</td>
<td>1393.47 / 1410.84</td>
<td>3938.6 / 3959.8</td>
</tr>
<tr>
<td>0.4</td>
<td>214.6 / 221.2</td>
<td>1368.75 / 1396.85</td>
<td>3909.2 / 3942.7</td>
</tr>
</tbody>
</table>

Table 3.3 Variation of 1\(^{st}\), 2\(^{nd}\), and 3\(^{rd}\) eigenfrequencies (rad.s\(^{-1}\)) of both planes for variable crack depths and for \(\phi_1 = \phi_{12} = 90^\circ\).
For a shaft of length \( L = 0.4 \) m, radius \( R = 0.004 \) m and with two cracks in positions \( L_1 = 0.1L \) and \( L_2 = 0.4L \), variable depths \( \bar{a}_1 = \bar{a}_2 = 0.2, 0.4, 0.5, 0.6, 0.8 \) and rotational angles \( \phi_1 = \phi_2 = 0^\circ \), the roots of characteristic determinant are presented in Table 3.3.

<table>
<thead>
<tr>
<th>Rotational angle ( \phi_1 = \phi_2 )</th>
<th>1(^{\text{st}}) eigenfrequency</th>
<th>2(^{\text{nd}}) eigenfrequency</th>
<th>3(^{\text{rd}}) eigenfrequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>223.9 / 225.76</td>
<td>1407.9 / 1415.56</td>
<td>3956.2 / 3965.65</td>
</tr>
<tr>
<td>30</td>
<td>224 / 225.79</td>
<td>1408.3 / 1415.68</td>
<td>3956.68 / 3965.82</td>
</tr>
<tr>
<td>60</td>
<td>224.85 / 225.89</td>
<td>1411.84 / 1416.07</td>
<td>3961.05 / 3966.3</td>
</tr>
<tr>
<td>90</td>
<td>225.73 / 226.03</td>
<td>1415.41 / 1416.63</td>
<td>3965.48 / 3967</td>
</tr>
<tr>
<td>120</td>
<td>225.95 / 226.07</td>
<td>1416.32 / 1416.8</td>
<td>3966.61 / 3967.2</td>
</tr>
<tr>
<td>150</td>
<td>226.09 / 226.09</td>
<td>1416.88 / 1416.88</td>
<td>3967.31 / 3967.31</td>
</tr>
<tr>
<td>180</td>
<td>226.09 / 226.09</td>
<td>1416.88 / 1416.88</td>
<td>3967.31 / 3967.31</td>
</tr>
</tbody>
</table>

Table 3.4 Variation of 1\(^{\text{st}}\), 2\(^{\text{nd}}\), and 3\(^{\text{rd}}\) eigenfrequencies (rad.s\(^{-1}\)) of both planes for variable crack rotational angle and for \( a_1 = a_2 = 0.4 \)

The eigenfrequency change of a shaft with two cracks of constant depth and variable rotational angle are examined. Suppose the shaft has two cracks with depths \( \bar{a}_1 = \bar{a}_2 = 0.4 \) and variable rotational angles \( \phi_1 = \phi_2 = 0^\circ, 30^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ, 180^\circ \). The eigenfrequency change is shown in Table 3.4. As the crack rotational angle approaches \( 180^\circ \), the crack is fully closed and remains closed during the normal mode vibration because of the vertical static load. Thus, the eigenfrequency approaches the corresponding value of the uncracked shaft in Table 3.4.

3.2.3 Calculation of the coupled response of the multi-cracked shaft.

The responses in the horizontal and vertical planes are given in Equations (3.31)-(3.36). The values of constants \( A_i, B_i \) with \( i = 1, 2, ..., 12 \) are obtained by the solution of the homogenous system in Equation (3.40).

\[
\begin{bmatrix}
A_1 & A_2 & \cdots & A_{12} & B_1 & B_2 & \cdots & B_{12}
\end{bmatrix}
\begin{bmatrix}
P_1 & 0 & \cdots & 0
\end{bmatrix}^T = \begin{bmatrix}
0 & 1 & \cdots & 0
\end{bmatrix}^T
\]

Supposing two cracks with depths \( \bar{a}_1 = \bar{a}_2 = 0.4 \) in positions \( L_1 = 0.1 \) and \( L_2 = 0.4 \) and rotational angles \( \phi_1 = \phi_2 = 0^\circ \), the dimensionless compliances take the following values:
When the vertical excitation in the clamped end exists, the system of Equation (3.40) becomes Equation (3.41).

\[
\begin{bmatrix} A_1 & A_2 & \cdots & A_{24} & B_1 & B_2 & \cdots & B_{24} \end{bmatrix}^T = \begin{bmatrix} 0.0001 & 0 & \cdots & 0 \end{bmatrix}^T
\]

(3.41)

The fact that \( \bar{c}_{45} = \bar{c}_{45} = 0 \) means that if a load is applied in plane 5 (the vertical plane), a coupling response does not exist in plane 4 (the horizontal plane).

The vertical load exists in this example by setting \( \bar{F}_i(0) = 0.0001 \), so after the system solution the variables \( B_j \) must equal zero in order to have no horizontal response. Consider matrix \( C \), the lower-left sub-matrix of \( P \) (see Appendix B). All of its terms contain the coupling dimensionless compliances \( \bar{c}_{45} \) and \( \bar{c}_{45} \), which are equal to zero. So, the terms with the zero compliance in the numerator are written off and matrix \( C \) is a zero matrix, indicating that there is no coupling from plane 5 to plane 4. When sub-matrix \( B \) is the zero matrix because of the zero values in compliances \( \bar{c}_{45} \) and \( \bar{c}_{45} \), then there is no coupling from plane 4 to plane 5. This happens when the cracks are totally closed. In this example, both cracks are symmetric in the vertical plane \( \phi_1 = \phi_2 = 0^\circ \) such that \( \bar{c}_{45} = \bar{c}_{45} = 0 \). Note that the frequency of the excitation changes from zero up to \( \omega = 5000 \text{ rad s}^{-1} \), indicating that the vibration passes through the first three resonances in the corresponding eigenfrequencies.

In Figure 3.23, the vertical response is shown while the horizontal response is zero for any excitation frequency. The resonance (maximum response) exists in the frequencies of Table 3.3. If both cracks are rotated equally such that \( \phi_1 = \phi_2 = 90^\circ \), the compliances calculated are equal to \( \bar{c}_{45} = \bar{c}_{45} = 0.10906 \), \( \bar{c}_{45} = \bar{c}_{45} = 0.0308 \), \( \bar{c}_{45} = \bar{c}_{45} = -0.11004 \), \( \bar{c}_{45} = \bar{c}_{45} = -0.00563 \).

All other characteristics (depth, position) remain the same as in the previous case. In this example, the coupling phenomenon is observed in all resonances of the spectrum. In Figure 8, the horizontal eigenfrequency intrudes into the vertical spectrum by adding a characteristic peak in the frequency of \( \omega = 226.02 \text{ rad s}^{-1} \), which is the horizontal eigenfrequency as shown in Table 3.2. The same goes for the horizontal spectrum where the vertical eigenfrequency intrudes into it and generates the additional peak in Figure 3.24 for \( \omega = 225.74 \text{ rad s}^{-1} \). The corresponding peaks are observed for any eigenfrequency, as shown in Figures 3.25 and 3.26. Tables 3.3 and 3.4 show an eigenfrequency shift of a few \( \text{ rad s}^{-1} \) and the practical
significance for condition monitoring or diagnosis does not come up. However, there are geometric shaft models of smaller slenderness ratio that can cause greater flexural moments in the cracked section, so as to generate a more intense crack effect and a greater frequency shift.

Figure 3.23 Vertical response at the free end of the cracked shaft as a function of excitation frequency.

Figure 3.24 a) Vertical and b) horizontal response in first resonance of Figure 2.6.
As the crack depth increases, the coupling phenomenon becomes stronger and the eigenfrequencies of the two planes further diverge. In continuing, the response in both planes is calculated for different rotational angles of the cracks. In order to observe which amplitude dominates the other, the horizontal and vertical amplitudes in the free end are divided to define the Amplitude Ratio \( (AR) \) as 
\[
AR = \left| \frac{Z_x(L)}{Y_y(L)} \right| \text{ with } AR \in [0,1)
\]

The Amplitude Ratio does take values near zero since the coupling effect cannot introduce vibrations of such amplitude as the amplitude of vibration in the plane of excitation. The Amplitude Ratio shows that for specific rotational angles, the horizontal vibration is zero, while in other rotational angles there is a generation of horizontal vibrations that remain at very low amplitude with respect to those of the vertical plane. Note that the excitation exists only in the vertical plane.

The vertical response does not significantly change during crack rotation; this is due to the fact that the excitation is in the same plane and so the crack breathing cannot affect a response that is due to an excitation in the same plane. On the other hand, the crack breathing produces a significant change in this response due to the coupling effect and not due to the excitation of this plane. The same phenomenon appears for the case of uniquely horizontal excitation. Figure 3.25 shows that for 5 different excitation frequencies, the maximum coupling always exists in the rotational angle of 90 degrees, which should be obvious because of the maximization of the coupling local compliance. Note that the excitation frequencies are chosen to be different from those near resonance, because the resonance event causes unpredictable variations of vibration amplitudes in both planes.

Another numerical example includes a clamped-free shaft of length \( L = 1\) m and radius \( R = 0.05\) m. The slenderness ratio \( 2L/R = 40 \) comparing to previous example offers higher eigenfrequencies and as result a greater absolute eigenfrequency shift due to crack. Each crack depth is \( a_1 = 0.8 \) and \( a_2 = 0.8 \). The cracks are located in the same point \( L_1 = L_2 = 0.1L \) in rotational angles \( \varphi_1 = 90^\circ \) and \( \varphi_2 = 270^\circ \). The local compliances for these characteristics are calculated as:

\[
\left[ \begin{array}{c}
\bar{\tau}_{\delta_5} \\
\bar{\tau}_{\delta_4}
\end{array} \right] = \left[ \begin{array}{c}
1.6590 \\
0.5530
\end{array} \right] , \quad \left[ \begin{array}{c}
\bar{\tau}_{\delta_6}
\end{array} \right] = \left[ \begin{array}{c}
-1.0045
\end{array} \right] , \quad \bar{\tau}_{\delta_4} = -0.08
\]

Both cracks are symmetrical to the center of the cracked cross section and the aim of this acceptance is to show that no coupling exists when the cracked section is symmetrical to the plane of the load even with two cracks present. The vertical response in Figure 3.26 has no additional peaks due to coupling while the horizontal response is zero.
If the second crack is moved in $L_2 = 0.2L$ then under the difference of bending moment in each cracked section, a coupling exists and the vertical and horizontal response obtain additional peaks as shown in Figure 3.27.

There is also a case in which the coupling exists one-way. To explain further, if one of the two coupling compliances $\overline{c}_{45}, \overline{c}_{54}$ is equal or even near zero then the coupling exists only from the one plane to the other. For example if $\overline{c}_{45} \rightarrow 0$ then the coupling exists from the horizontal to the vertical plane while the coupling in the inverse direction is not observed. Also if $\overline{c}_{54} \rightarrow 0$ then only the coupling from the vertical to the horizontal plane is presented. For $a_1 / R = a_2 / R = 0.4$ there are crack rotational angles such as $\phi_1 = \phi_2 = 130^\circ$ that give compliance values: $\overline{c}_{45} = \overline{c}_{54} = 0.01704$, $\overline{c}_{46} = \overline{c}_{56} = 0.00315$, $\overline{c}_{44} = \overline{c}_{55} = -0.0279$, $\overline{c}_{45} = \overline{c}_{54} = 0$.

The frequency response for crack positions $L_1 = 0.1L$ and $L_2 = 0.2L$ obtains double peaks only in horizontal plane while in vertical plane no coupling is introduced as it is shown in Figure 3.28.

![Figure 3.25 Amplitude Ratio AR as a function of Rotational angle for 5 different values of excitation frequency](image)

*Figure 3.25 Amplitude Ratio AR as a function of Rotational angle for 5 different values of excitation frequency (dash: $\omega = 10 \text{ rad/sec}$, continuous: $\omega = 90 \text{ rad/sec}$, short dash: $\omega = 250 \text{ rad/sec}$, dash dot: $\omega = 500 \text{ rad/sec}$, dash dot dot: $\omega = 800 \text{ rad/sec}$).*
Figure 3.26 Vertical response of the shaft when the cracks are located symmetrically to the vertical plane and at the same location. $a_1 / R = a_2 / R = 0.8 \quad L_1 = L_2 = 0.1L, \quad \phi_1 = 90, \quad \phi_2 = 270^\circ$.

Figure 3.27 a) Vertical and b) horizontal response of the shaft when cracks are located symmetrically to the vertical plane and at different location. $a_1 / R = a_2 / R = 0.8 \quad L_1 = 0.1L, \quad L_2 = 0.2L, \quad \phi_1 = 90, \quad \phi_2 = 270^\circ$.
Figure 3.28 The first resonance in vertical (continuous line) and horizontal (dashed line) plane for the case that \( c_{44} = c_{44} = 0 \). Coupling exists only in the horizontal plane.

3.2.4 Experimental procedure.

In order to observe and validate the theoretical model of the coupled bending vibrations, an experiment was carried out. A clamped-free shaft with one cut was vibrated using a vertical excitation that is transferred to the clamped end of the shaft using the base shown in Figure 3.29.

The excitation frequency was a function of time \( \omega(t) = 10t \), \( 0 < t < 500s \) with a maximum value of 5000 Hz so as to pass through the seventh resonance frequency of the beam. Using an accelerometer in the free end of the beam, two signals, the vertical and the horizontal accelerations were acquired in the free end. The accelerometer had an insignificant weight.
with respect to the shaft, and so it was not accounted for in the analytical procedure. The acquired signals for vertical and horizontal acceleration in the free end of the uncut shaft are shown in Figure 3.30.

![Figure 3.30](image)

*Figure 3.30 Experimental measurements of the a) horizontal and b) vertical acceleration at the free end of the intact shaft as a function of excitation frequency.*

A crack is very different from a slot [201] but the comparison between the analytical results of the cracked shaft and the experimental results of a slotted shaft can be justified, because cracks as well as slots cause coupling at some rotational angles. This procedure focused on investigating the response due to coupling that is provoked either from cracks or slots, so long as greater depths in both defects intensively affect the coupling. An eigenfrequency shift between the two cases is prospective.

The basic difference between a cracked and a slotted shaft is that the slotted does not “breathe” and remains open during the rotation. A second difference is that the stress intensity factor of the slotted shaft should be different from that of the crack, because the radius of the crack tip tends to zero since the radius of the slot depends on the radius of the saw used to open the slot. The calculation of the stress intensity factor depends on the applied loads and the geometry (depth and width) of the crack.

The common property that both slotted and cracked shaft have, is the shifting of the centre of shear stresses, from the centre of the circular section to a rather asymmetric place, on a perpendicular to the crack edge at its middle. This physically causes the coupling and this is common in both configurations.

A Fast Fourier Transform gave the acceleration as a function of the flexural frequency of the shaft, and the resonance frequencies were calculated from the FFT plot. The resonance
frequencies for the uncut shaft are shown in Table 3.6. Note that, until then, the horizontal response was due to geometric asymmetry of the experimental model to the plane of the excitation, which was the vertical plane. The vibrator machine could not vibrate perfectly in the vertical direction due to manufacturing constraints. The same applied for the model of the beam and the base in Figure 3.31, in which the welding joints made the construction not exclusively symmetric to the vertical plane. So, measures of horizontal acceleration were expected even without a cut. The differences between the analytical (Table 3.5) and the experimental (Table 3.6) eigenfrequencies for uncracked shaft \( a/D = 0 \) exist because the supposedly clamped end was not exclusively clamped due to the welding joint. Also, the base could not be so stiff as to exclude all horizontal revolution when it was vibrated. The torsion in the clamped end generated a revolution of the base in the vertical plane as shown in Figure 3.29. These phenomena could be modeled using torsional springs (vertical and horizontal) in the clamped end as shown in Figure 3.31.

In this case the boundary conditions in the clamped end become

\[
Y''_r(0) - K_rY'_r(0) = 0 \\
Z''_r(0) - K_rZ'_r(0) = 0
\]

and \( K_r \) is the stiffness of the torsional spring in the vertical and horizontal direction. When \( K_r \) takes values towards infinity the model inclines to the analytical one. With the new boundary conditions, the characteristic determinant \( \det(P) \) is a function of frequency \( \omega \) and of the stiffness \( K_r \). It is feasible to estimate the values of \( K_r \) so as to match the analytical eigenfrequencies to the experimental resonance frequencies for the case of \( a/D = 0 \). The FFT plot of the intact experimental shaft is shown in Figure 3.32. The first three frequencies of the resonance in both directions are presented in Table 3.6.

By setting in matrix \( P \) all local compliances equal to zero (no crack), and \( \omega = 188.49 \text{ rad s}^{-1} \) (first experimental resonance frequency) the equation \( \det(P)|_{\omega=188.49} = 0 \) gives the value of \( K_r = 962.2 \text{ Nm rad}^{-1} \). In Figure 3.34 the vertical response is plotted as a function of...
frequency and in Table 3.4 the first three eigenfrequencies are presented while the horizontal response of the uncracked shaft is zero.

<table>
<thead>
<tr>
<th>Crack Depth</th>
<th>$a / D = 0$</th>
<th>$a / D = 0.1$</th>
<th>$a / D = 0.2$</th>
<th>$a / D = 0.4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak</td>
<td>$1^{st} / 2^{nd}$</td>
<td>$1^{st} / 2^{nd}$</td>
<td>$1^{st} / 2^{nd}$</td>
<td>$1^{st} / 2^{nd}$</td>
</tr>
<tr>
<td>Vertical Plane</td>
<td>1</td>
<td>30.0 / -</td>
<td>30.0 / 30.0</td>
<td>29.98 / 29.99</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>196.71 / -</td>
<td>196.71 / 196.7</td>
<td>196.71 / 196.7</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>565.64 / -</td>
<td>565.64 / 565.64</td>
<td>565.33 / 565.66</td>
</tr>
<tr>
<td>Horizontal Plane</td>
<td>1</td>
<td>- / 30.0</td>
<td>30.0 / 30.0</td>
<td>29.98 / 30.0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>- / 196.71</td>
<td>196.71 / 196.7</td>
<td>196.71 / 196.7</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>- / 565.64</td>
<td>565.64 / 565.64</td>
<td>565.33 / 565.66</td>
</tr>
</tbody>
</table>

*Table 3.5* $1^{st}$, $2^{nd}$ and $3^{rd}$ eigenfrequency variation (Hz) as a function of crack depth calculated in the model

In the shaft of Figure 3.29, a cut was made with variable dimensionless depth $\bar{a} = 0.2, 0.4, 0.8$ and $\phi = 90^\circ$; the signals for vertical and horizontal acceleration were acquired. In Figures 3.32 and 3.33 the FFTs for every response was plotted for each corresponding cut depth.

The analytical model considers one crack so as to remain comparable with the experimental one (one cut). In the mathematical model, then, one crack is at $L_1 = 0.2L$ and the other one is at the free end, where $L_2 = 1L$, so as to have no effect on the vibration. The frequency response is calculated for variable crack depths $\bar{a} = 0.2, 0.4, 0.8$ and $\phi = 90^\circ$. Note that in order for the coupling peaks to become visible, the excitation frequency changes with a step of $10^{-9}$ in the regions where coupling exists. In the other regions of excitation frequency, the step is equal to $10^{-3}$; this is obligatory for procedure endurance. In Figure 3.34, the frequency response is plotted for each corresponding crack depth.

Comparing the first three resonance frequencies calculated from the FFTs in Figures 3.32, 3.33, and 3.34 in Table 3.6, it can be seen that as the depth of the cut increases, the resonance frequencies in the vertical direction decrease. This is expected because the deeper cut decreases the shaft stiffness. Note that in the case of the uncracked shaft, the additional peak of the horizontal response in the vertical response is quite faint, but as the
crack depth increases the additional peak from one direction to the other becomes clearly visible and of bigger amplitude.

<table>
<thead>
<tr>
<th>Cut Depth</th>
<th>Peak</th>
<th>1st / 2nd</th>
<th>1st / 2nd</th>
<th>1st / 2nd</th>
<th>1st / 2nd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical</td>
<td>1</td>
<td>30.0/30.0</td>
<td>29.5/29.5</td>
<td>30.0/30.0</td>
<td>29.8/29.8</td>
</tr>
<tr>
<td>Plane</td>
<td>2</td>
<td>192.2/196.9</td>
<td>191.5/196.5</td>
<td>190.7/195.2</td>
<td>189.5/193.9</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>538.5/547.4</td>
<td>537.8/546.4</td>
<td>537.6/546.1</td>
<td>536.5/545.6</td>
</tr>
<tr>
<td>Horizontal</td>
<td>1</td>
<td>30.0/30.0</td>
<td>29.5/29.5</td>
<td>29.6/29.6</td>
<td>29.6/29.6</td>
</tr>
<tr>
<td>Plane</td>
<td>2</td>
<td>192.3/196.8</td>
<td>191.6/196.4</td>
<td>190.8/195.1</td>
<td>189.6/193.8</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>538.5/546.1</td>
<td>538.4/545.8</td>
<td>537.7/545.5</td>
<td>536.8/545.1</td>
</tr>
</tbody>
</table>

Table 3.6 1st, 2nd and 3rd eigenfrequency variation (Hz) as a function of cut depth measured in the experiment

This coupling phenomenon becomes clear when two neighboring eigenfrequencies exist in the area of the resonance of each direction. Note that exactly equal eigenfrequencies are observed in both planes. The corresponding analytical responses show the same phenomenon. The uncracked shaft has no horizontal response and, in the vertical response, the observed coupling is stronger as the crack depth increases.

Figure 3.32 FFT for (a) vertical and (b) horizontal acceleration of the free end of the shaft without cut in the region of 0–600 Hz.

The change in analytical eigenfrequencies does not exactly match those of the experimental calculations; this is due to the fact that the dimensionless compliances are computed for the
semi-closed crack at the rotational angle of 90 degrees, but the experimental cut does not follow this rule because it is made with a 0.5 mm saw (cut). So, the local compliances in the experiment are much different than those calculated from the stress intensity factors. Besides the slenderness ratio of a cracked beam is a parameter that affects the reduction of the eigenfrequencies. Bigger slenderness ratios cause smaller eigenvalues reduction and vice versa [202]. Here the shaft used had a slenderness ratio \( \lambda = L / (R / 2) = 2L / R = 2 * 400 / 4 = 200 \) that is a rather big one and the reduction of the eigenvalues are not expected to be large. The experimental results are generally repeatable inside a small range of discrepancy.

The response amplitude for various excitation frequencies in the area of first three resonances is calculated for both planes in Figure 3.35 for various values of rotational angles of the crack.

![Figure 3.33 FFT for (a) vertical and (b) horizontal acceleration of the free end of the cut shaft with a cut of \( a=0.8 \) in the region of 0–600 Hz.](image)

![Figure 3.34 (a) Vertical and (b) horizontal response of the free end of the cracked shaft with a crack of \( a=0.8 \).](image)
As it can be seen in Figure 3.35a the vertical response amplitude does not change as the crack obtains different values while in Figure 3.35b it can be seen that the horizontal
response amplitude obtains its maximum value for the case of rotational angle in 90 degrees proving that the coupling is more intense when the coupling compliance obtains its maximum (see also Figure 2.19). The experimental measurement of amplitude response as shown in Figure 3.30 proves, after a Short Time Fourier Transform, that the vertical amplitude during 2\textsuperscript{nd} resonance does not change as the cut depth increases (see Figure 3.36) while the horizontal amplitude is clearly more sensitive in increment as the cut depth increases.

### 3.2.5 Conclusions for the coupled vibrations of the cracked beam

In this investigation of coupled bending vibrations, there were two novel points. The first was the different approach of the local compliance matrix during the rotation of the cracks. The four terms of the local compliance matrix were defined supposing that the crack breathes under the effect of gravity. The second significant point was the modeling of bending coupling due to the crack. It was proven that the crack provoked a response in a different direction than that of the excitation. Furthermore, the coupled response in the horizontal plane under vertical excitation is maximized when the crack is rotated by 90 degrees in reference to the vertical plane. This is the result of the value of the local coupled compliance, which is maximized at this rotational angle. The fact that two cracks are considered in this chapter comes up due to a real situation of high bending moments in a shaft at two different points as in the case of four points bending of beams. The frequency response in the analytical and experimental procedures proved that in each spectrum there are resonances in the eigenfrequencies of both planes. In the vertical spectrum, both vertical and horizontal resonances are presented; the same is valid for horizontal spectrum. The experimental procedure proved that the deeper crack makes the coupling phenomenon more intense. The signature of coupling, the “double peak” in the spectrum, appears more clearly in both spectrums as the crack depth increases.

The analytical and experimental procedures do not agree on the eigenfrequency values because the analytical procedure considers a crack while the experimental considers a saw cut that induces a different local compliance. Also, there are manufacturing defects in the experimental model that do not introduce exactly equal stiffness between the two planes. This results in different eigenfrequencies for the two planes even in the case of the uncracked shaft. The virtual spring that was located on the clamped end can bring the experimental measurements into agreement with the analytical ones, but the spring stiffness calculation must be made for every eigenfrequency (first, second, third etc) in order to equilibrate the experimental and analytical eigenfrequencies. It is assumed that the spring stiffness is constant for a range of excitation frequencies. In conclusion, it has clearly been proven that a
crack induces a coupling in bending vibrations and that the phenomenon becomes stronger as the crack depth increases. The analysis presented in this chapter has accurately yielded the coupled vibration characteristics of a non-rotating shaft with two cracks, thereby adding a different vibration model to the “family” of coupled vibrations that has been extensively studied in recent years.
3.3 A continuous model approach for the rotating cracked shaft

In this chapter, the cross-coupled bending vibrations of a cracked rotating shaft mounted in resilient bearings are investigated. The equations of motion of the continuum and isotropic rotating model of the shaft follow the theory of Rayleigh. The governing equations are coupled in the two main directions, and the partial solution is obtained by solving a linear system of equations. The coupling is introduced in three different ways: the equations of motion, the resilient bearings and the crack. A main focus is made in the introduction of coupling due to crack compliance variance while rotation with the cross-coupling terms of the local compliance matrix due to the crack to be calculated analytically as functions of the rotational angle. The three causes of coupling between the vertical and horizontal vibrations should be distinguished with regard to the effects that each one of them has on the dynamic response of the rotor. Inversely, the existence of each type of coupling in the frequency response could be used to identify the respective cause.

3.3.1 Equation of motion

In this simulation of the motion of the rotating shaft, the rotary inertia, the shear deformation, the torque of power transmission and the gyroscopic effect are taken into consideration. Assume a uniform homogenous and cracked rotating Timoshenko shaft (Figure 3.35) of Young's modulus of elasticity $E = 205.8$ GPa, shear modulus $G = 79.76$ GPa, moment of inertia of the cross-section about $X$ axis $I = 3.06 \times 10^{-7}$ m$^4$, mass density $\rho = 7860$ kg m$^{-3}$, shear factor $k = 10/9$ for circular cross-section, length $L = 2$ m, radius of cross-section $R = 0.025$ m, radius of gyration $r = 0.0125$ m, and Poisson ratio $\nu = 0.29$. The shaft is rotating with a rotational speed $\Omega$, whirling with a frequency $\omega$, and transmitting a power with an axial torque $T$. Also consider a transversely located disk in the mid-span ($x = L / 2$) of the shaft of the same material, with radius $R_d = 0.15$ m, mass $m_d = 27.78$ kg, and thickness $L_d = 0.05$ m. The breathing crack exists in the mid-span, just next to the disk. If $Y(x,t)$ and $Z(x,t)$ are the vertical and horizontal response of the axial coordinate $x$ and time $t$, respectively, then by supposing the complex notation $U_j(x,t) = Y_j(x,t) + i Z_j(x,t)$, the coupled governing equation of motion is written as [203]: where $j = 1$ for the first part of the shaft from the left end up to the crack and $j = 2$ for the part from the crack up to the right end (Figure 3.35). Equation (3.42) is a fourth order complex partial differential equation that has a solution of the form in Equation (3.43). Substitution of Equation (3.43) into Equation (3.42) yields the complex characteristic Equation (3.44), a fourth degree polynomial equation of $\lambda_j$, 

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Figure 3.35 The model of the cracked shaft.

\[ EI \frac{d^4 U_j}{dx^4} - iT \frac{d^4 U_j}{dx^4} - \left( \frac{EI \rho}{kG} + \rho A r_0^2 \right) \frac{d^4 U_j}{dx^4} + 2i \rho A r_0^2 \frac{\partial^4 U_j}{\partial x^4} t + \frac{\rho^2 A r_0^2}{kG} \frac{\partial^4 U_j}{\partial x^4} + 2i \rho A r_0^2 \frac{\partial^4 U_j}{\partial x^4} t + 2i \rho A r_0^2 \frac{\partial^4 U_j}{\partial x^4} t = 0 \] (3.42)

\[ U_j(x,t) = u_j(x)e^{j\omega t} \] (3.43)

\[ \omega^2 \left( -iT \lambda_j + \rho \left( EI \lambda_j + Ar_0^2 \rho \omega (\omega - 2\Omega) \right) \right) + kG \left( -A \rho \omega^2 + \lambda_j^2 \left( -iT \lambda_j + EI \lambda_j + Ar_0^2 \rho \omega (\omega - 2\Omega) \right) \right) = 0 \] (3.44)

Two of the roots of (3.44) are complex numbers; meanwhile, the other two are imaginary. Actually the two imaginary roots are complex with near zero real parts. By setting the roots as \( \lambda_{j,1}, \lambda_{j,2}, \lambda_{j,3}, \lambda_{j,4} \), the partial solution becomes as in Equation (3.45)

\[ u_j(x) = q_{j,1}e^{j\lambda_{j,1}x} + q_{j,2}e^{j\lambda_{j,2}x} + q_{j,3}e^{j\lambda_{j,3}x} + q_{j,4}e^{j\lambda_{j,4}x}, \ j = 1,2 \] (3.45)

Coefficients \( q_{j,1}, q_{j,2}, q_{j,3}, \) and \( q_{j,4} \), are also complex numbers that are defined using the boundary conditions for displacement, slope, bending moment, and shearing force, which in a Timoshenko shaft with gyroscopic effect are defined as in [204-206],

Displacement: \( U_j(x,t) \)

Slope: \( \Theta_j(x,t) = \frac{\partial U_j(x,t)}{\partial x} \)
3.3.2 Boundary conditions

The cracked rotor is assumed to be mounted in hinged supports at both ends. The disk and the crack are assumed to be in the mid-span \((L_1 = L / 2 = 1 \text{ m})\) with the crack in the right side of the disk. Also, \(I_p = 0.5m_d R_p^2 = 0.31252 \text{ kg m}^2\) and \(I_c = 0.25m_d R_d^2 + 1/12m_d L_d^2 = 0.162 \text{ kg m}^2\) are the mass polar moments of inertia about axis \(Ox\) and diametric axis of the disk respectively (see Figure 3.35); \(\Theta_j(x,t)\) and \(\Theta_k(x,t)\) are the conjugate complex numbers of \(\Theta_j(x,t)\) and \(\Theta_k(x,t)\), respectively. For this specific problem, \(m_u = 0.01 \text{ kg}\), \(r_u = 0.1 \text{ m}\), and \(\phi_u\) are the unbalance mass, the distance from the centre of the rotation, and the angle between the vector of unbalance and the horizontal axis, respectively. For simplicity, \(r_u\) is a constant distance and \(\phi_u = 0\). Due to the fact that the excitation of the system is the unbalance force, the whirling of the shaft is synchronous with the rotation, which means \(\omega = \Omega\). Therefore the boundary condition (BC) of shearing force, for the case of a disk with and without unbalance, becomes as in Equations (3.46) and (3.47), respectively.

\[
V_z(L_1,t) = V_z(L_1,t) - \left( m_u \omega^2 \right) U_1(L_1,t) + m_u r_u^2 \omega^2 e^{i\omega t} \quad (3.46)
\]

\[
V_z(L_1,t) = V_z(L_1,t) - \left( m_u \omega^2 \right) U_1(L_1,t) \quad (3.47)
\]

At both ends, the displacement and the bending moment are equal to zero:

\[
U_1(0,t) = U_1(L,t) = 0 \quad (3.48)
\]

\[
M_1(0,t) = M_1(L,t) = 0 \quad (3.49)
\]

In the mid-span:

a) The bending moment at the left side of the crack (the same as the right side of the disk) results in a slope discontinuity in the two parts of the shaft which is described by Equation (3.50):

\[
(\epsilon + i\delta) M_1(L_1,t) + (\beta + i\gamma) M_1(L_1,t) = (\Theta_z(L_1,t) - \Theta_1(L_1,t)) \quad (3.50)
\]

where \(\epsilon = 0.5(c_{35} + c_{44})\), \(\beta = 0.5(c_{35} - c_{44})\), \(\gamma = 0.5(c_{45} + c_{54})\), and \(\delta = 0.5(c_{45} - c_{54})\).
b) The bending moment on both sides of the disk follows the equation:

\[ M_2(L_1,t) = M_1(L_1,t) + i \Theta(L_1,t)(I_p \Omega - I_c \dot{\omega}) \]  

(3.51)

c) The displacements at both sides of the disk are equal:

\[ U_1(L_1,t) = U_2(L_1,t) \]  

(3.52)

When the crack is closed the local compliances become zero and the boundary condition of Equation (3.50) is transformed into Equation (3.53), describing the continuity in slopes at both sides of the disk:

\[ \Theta_1(L_1,t) = \Theta_2(L_1,t) \]  

(3.53)

### 3.3.3 Calculation of gravity response

As mentioned above, the crack breathes due to a gravity effect in the elastodynamic behavior of the shaft. The gravity response is assumed to surpass the unbalance response so as to set the crack condition (breathing). In theory, this happens when the rotational speed is not near the critical speed, but in practice the crack breathing due to gravity can be observed at every speed under special geometric characteristics of large rotating machines. In this work, the gravity is assumed to be static (independent of time) and the gravity response \( U_{g_1}(x) \) is obtained by the ordinary differential Equation (3.54) as in [207]:

\[ EI \frac{d^4 U_{g_1}(x)}{dx^4} - iT \frac{d^3 U_{g_1}(x)}{dx^3} = -\rho A g, \ i = 1, 2 \]  

(3.54)

By neglecting the torque for simplicity, and for hinged-hinged boundary conditions with the disk at the mid-span, the solution for the two parts of the shaft becomes,

\[ U_{g_1}(x) = -\frac{\rho A g}{24EI} x^4 + \frac{g m_x + 2AgL \rho}{12EI} x^3 - \frac{3gL^2 m_x + 4AgL^2 \rho}{12EI} x \]  

(3.55)

for \( 0 \leq x \leq L/2 \)

\[ U_{g_2}(x) = -\frac{\rho A g}{24EI} x^4 - \frac{g m_x - 2AgL \rho}{12EI} x^3 + \frac{gL m_x}{2EI} x^2 - \frac{9gL^2 m_x + 4AgL^2 \rho}{12EI} x + \frac{gL^3 m_x}{6EI} \]  

(3.56)

for \( L/2 \leq x \leq L \)
3.3.4 Calculation of critical speeds

In order to present the effect of coupling at critical speeds, the cracked and the uncracked models are analyzed. When no crack exists, the boundary condition of Equation (3.53) is substituted by Equation (3.50). By substituting the general solution of Equation (3.57)

\[ U_j(x,t) = (q_{j,1} e^{j_1 t} + q_{j,2} e^{j_2 t} + q_{j,3} e^{j_3 t} + q_{j,4} e^{j_4 t}) e^{j_0 t} \] (3.57)

into the eight boundary conditions, Equation (3.46) - (3.53), the homogenous system \((8 \times 8)\) for the variables \(q_{j,1}, q_{j,2}, q_{j,3}, q_{j,4}\) for \(j = 1, 2\) of Equation (3.58) is obtained,

\[
[P] \begin{bmatrix} q_{1,1} & q_{1,2} & q_{1,3} & q_{1,4} & q_{2,1} & q_{2,2} & q_{2,3} & q_{2,4} \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & \cdots & 0 \end{bmatrix}^T \] (3.58)

The real and imaginary roots of \(|P| = 0\) for \(\omega = \Omega\) are the critical speeds. The roots of real part of characteristic equation are the vertical critical speeds, and the roots of the imaginary part of the characteristic equation are the critical speeds corresponding to the horizontal plane. For the cracked model, the boundary condition of Equation (3.50) introduces the parameter of time into the model. Time is a factor that specifies the crack local compliance in accordance with the breathing condition. This fact makes the characteristic determinant also a function of time. For example, if \(t = \frac{1}{4} (2\pi / \Omega)\), the crack is semi-closed and the local compliances have different values from those of any other value of time in the same period.

![Figure 3.36](image)

Figure 3.36 (a) Real and (b) imaginary part of the characteristic determinant as a function of \(\Omega\).

As shown in Figure 2.18, there is no time value for which all compliances are equal to each other, except if \(t \in \left[ \frac{-\phi_c}{\Omega}, \frac{\phi_c}{\Omega} \right]\) where \(\phi_c = 36^\circ\) is the angle of rotation in which a crack of \(\bar{a} = 0.4\) starts to open. The characteristic determinant is calculated for a specific value of
time (i.e in the ¼ of period), and plotted as real parts in Figure 3.36a and as imaginary parts in Figure 3.36b as a function of \( \Omega / \omega_0 \). The value \( \omega_0 \) is used in this chapter to express frequency in dimensionless form and is defined as \( \omega_0 = \left( EI / \rho AL^4 \right)^{1/2} \). For the geometry of this section \( \omega_0 \approx 16 \). The zero crossing values express the eigenfrequencies for the instant form of the crack (here the crack is instantaneous semi-open in the ¼ of period) and the logarithmic magnitude is used here so as to achieve the visibility of function progress since in areas near roots it tends rapidly from \(-\infty\) to \(+\infty\).

### 3.3.5 Calculation of the frequency response

The response in the frequency domain is calculated by an iterative procedure. In each iteration, a rotating/whirling frequency value is defined and the time response for one period of time, in steps of \( \Delta t = 0.01(2 \pi / \Omega) \) s, is calculated. In each time response, the amplitude is measured and diagrams of amplitude as a function of frequency are obtained. The cases for the uncracked and cracked shaft are investigated. The procedure is repeated for a frequency range higher than the third critical speed with a frequency step change of \( \Delta \Omega = 1 \) rad s\(^{-1}\), so as to define the frequency areas of critical speeds (see Figure 3.37), and then a step change of \( \Delta \Omega = 0.001 \) rad/s is used in order to examine the effect of coupling from both sides of first critical speed (see Figure 3.38). Note that the resonance in the 2\(^{nd}\) critical speed does not appear in this response because the measurement is taken in the mid-span, which is a nodal point of the second eigenmode. For simplicity, only forward whirling will be examined.

![Figure 3.37](image_url)  
*Figure 3.37 (a) Vertical and (b) horizontal amplitude as a function of rotating/whirling speed.*
An obvious difference in amplitude change is observed in Figure 3.38 when comparing the cases of uncracked and cracked shafts. The amplitude of the uncracked shaft has one peak in the first critical speed and this also applies for horizontal and vertical amplitude in the same rotating frequency, as long as the system is isotropic. When the crack is introduced, each resonance has two peaks at a lower rotating speed than that of the uncracked one. The resonance frequencies that correspond to the vertical amplitude are a bit smaller compared to those of the horizontal amplitude. This is due to the local stiffness of the crack, which always remains larger in the horizontal plane than in the vertical. These additional peaks are observed in both planes at the same value of the rotating frequency because of the coupling. Note that the magnitude is larger for smaller $\Delta \Omega$ and $\Delta t$.

![Figure 3.38. (a) Vertical and (b) horizontal amplitude as a function of rotating/whirling speed near first critical speed.](image)

It is a fact that the critical speed shift (cracked / uncracked) calculated from frequency response of Figure 3.38 is miniscule and may not be noticed in practice. However the case where the compliance coupling terms are neglected, provides a greater frequency shift since the coupling compliance is not always an additional magnitude in the direct compliance, with this phenomenon to exist due to their signs variation as positive and negative as many researchers have presented [208]. Additionally a recent experimental work in cracked beam [209] provides the miniscule eigenfrequency shift when the compliance coupling terms are in their large values (crack orientation in $\varphi = 90^\circ$; see Figure 2.18). On the other hand an appropriate time-frequency decomposition using wavelet transform encourage this miniscule eigenfrequency shift notification since this decomposition can localize it's components in such a "narrow" time domain between the double peak in the frequency response.
3.3.6 Numerical example 1

Under an unbalance excitation of 0.001 kg m, the cracked rotor vibrates with the effect of coupling. When an unbalance exists, the general solution of Equation (3.45) is substituted in the boundary condition of Equation (3.46) instead of that of Equation (3.47), and the following non-homogenous (8x8) system of Equation (3.59) is obtained,

\[
\begin{bmatrix}
q_{1,1} & q_{1,2} & q_{1,3} & q_{1,4} & q_{2,2} & q_{2,3} & q_{2,4} & q_{2,5}
\end{bmatrix}^T
= \begin{bmatrix}
0 & 0 & 0 & 0 & \ldots & 0 & m_2 r_2 \gamma
\end{bmatrix}^T
\]

The cracked and uncracked cases are compared. For each case, the forward synchronous (\( \Omega = \omega \)) whirling time response is calculated in the mid-span for both planes of vibration and for rotational speeds different from the first critical, in order to avoid singularities. The rotational speed is set to \( \Omega = 5.625 \) (see Figure 3.39), \( \Omega = 5.831 \) (a speed very close to critical (see Figure 3.40)), and \( \Omega = 5.937 \) (a speed after the first critical (Figure 3.41)). It is clear that the response due to coupling is observed for crack rotational angles near 90, 180 and 270 degrees or \( \frac{1}{4}, \frac{1}{2} \) and \( \frac{3}{4} \) of the time period. In these angles of rotation, the coupling terms \( \tilde{c}_{54} \) and \( \tilde{c}_{45} \) get the higher absolute values. This difference due to coupling has been also observed by Darpe [210] in 2002. The opening of the crack permits an additional response in the vertical direction near 180 degrees, but also an additional response near 90 and 270 degrees as shown in Figures 3.39, 3.40 and 3.41.

![Figure 3.39](image)

*Figure 3.39 (a) Vertical and (b) horizontal time response in mid-span for the two cases, in rotational speed \( \Omega = \omega = 90.0 \) rad/s.*
The time response, treated as a signal, can be analyzed in its components with the Fast Fourier Transform (FFT) in order to more clearly observe the effect of the coupling in additional harmonic development. Each of the signals of Figure 3.40 and 3.41 are transformed using FFT and the respective results are shown in Figure 3.42 and 3.43. The component of $\Omega = 5.625$ rad/s, is synchronous with the rotational / whirling frequency and, as a result, has the highest amplitude. Also, the components with frequencies of 2/rev, 3/rev, 4/rev and more are seen to have lower amplitudes as the frequency increases [211]. It is known that a crack introduces such harmonics as in [212], but in the case where the crack introduces a coupling, these harmonics have higher amplitudes.

The coupling effect and the additional harmonic development become more intense as the vibration amplitude increases and this happens as the system gets near to resonance. At these speeds the coupling phenomenon provokes additional components with their amplitude is greater with respect to this of synchronous component than this of cases far from resonance. The domination of synchronous response is of course observed but the relative amplitude becomes greater.
The same results are also verified in the signals of Figure 3.40, (see Figure 3.43) where the highest amplitude is observed at the synchronous frequency, while the other components appear in higher harmonics with lower amplitude. The FFT of signals for \( \Omega = 5.937 \), in Figure 3.41, are judged as not necessary for plotting such as those of Figure 3.39. Note that at the speed of \( \Omega = 5.937 \) there is a phase inversion in the time domain response. A general conclusion derived from spectra given in Figures 3.42 and 3.43 is that since the coupling is introduced mainly two times in the period duration the 2Xrev harmonic is mainly amplified. The development / amplification of the 2Xrev harmonic is of course the main result of the crack breathing no matter what modeling of breathing is used but the co-existence of coupling increases the amplification / development of 2Xrev.

Figure 3.42 FFT of (a) vertical and (b) horizontal response for \( \Omega = \omega = 90.0 \) rad/s.
Figure 3.43 FFT of (a) horizontal and (b) vertical response for $\Omega = \omega = 93.3$ rad/s.
3.4 The resilient bearing model

In the previous numerical example, the bearing was assumed to be rigid in order to focus on the crack effect. The coupling introduced by the crack is presented in the same way as the coupling due to the cross-coupled bearing stiffness and damping coefficients. This is also shown in the frequency response domain by double peaks. In this numerical example, a cracked rotor model in resilient bearings is analyzed, and results for frequency response are obtained. In Figure 3.44, the rotor-bearing system characteristics are shown, as given by Hong [213] for comparison purposes.

A shaft of diameter $D = 25$ mm, length $L = 1200$ mm, made of steel with $E = 200$ GPa and density $\rho = 8000$ kg m$^{-3}$, is supported by two bearings at its two ends, with the following dynamic properties,

$$
K_k = \begin{bmatrix} k_{yy} & k_{yx} \\ k_{yx} & k_{xx} \end{bmatrix} = \begin{bmatrix} 1.2 \times 10^6 & 0 \\ 0 & 1.2 \times 10^6 \end{bmatrix} \text{N m}^{-1}, \ C_k = \begin{bmatrix} c_{yy} & c_{yx} \\ c_{yx} & c_{xx} \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} \text{N s m}^{-1}
$$

![Figure 3.44 The resilient bearing model.](image)

![Figure 3.45 Left journal amplitude (continuous line: uncracked, dotted line: cracked 40% of the radius).](image)
In this model, the boundary conditions at both ends of the shaft are calculated in complex form by combining the boundary conditions for the spring and damper given as in (3.60) and (3.61).

The forces applied by the bearing are proportional to the displacement $U_{ij}(x,t) = U_{ij}(x) + i Z_{ij}(x)$, as well as to its first time derivative. Finally, the two boundary conditions take the form as in Equations (3.60) and (3.61).

$$V_i(0,t) = (s_i - is_i) U_i(0,t) + (s_i + is_i) \dot{U}_i(0,t) + \left( p_i - ip_i \right) \frac{\partial}{\partial t} U_i(0,t) + \left( p_i + ip_i \right) \frac{\partial}{\partial t} \dot{U}_i(0,t)$$  (3.60)
\[ V_z(L,t) = \left( s_1 - is_4 \right) U_z(L,t) + \left( s_2 + is_3 \right) \tilde{U}_z(L,t) + \left( p_1 - ip_4 \right) \frac{\partial}{\partial t} U_z(L,t) + \left( p_2 + ip_3 \right) \frac{\partial}{\partial t} \tilde{U}_z(L,t) \] (3.61)

\[ s_1 = 0.5 \left( k_{yy} + k_{zz} \right), \quad s_2 = 0.5 \left( k_{yy} - k_{zz} \right), \quad s_3 = 0.5 \left( k_{yz} + k_{zy} \right), \quad s_4 = 0.5 \left( k_{zy} - k_{yz} \right) \] (3.62)

\[ p_1 = 0.5 \left( c_{yy} + c_{zz} \right), \quad p_2 = 0.5 \left( c_{yy} - c_{zz} \right), \quad p_3 = 0.5 \left( c_{yz} + c_{zy} \right), \quad p_4 = 0.5 \left( c_{zy} - c_{yz} \right) \] (3.63)

\[ \tilde{U}_i(x,t) = Y_i(x,t) - itZ_j(x,t) \] is the complex conjugate value of the displacement \( U_i(x,t) \).

In the present case, where an unbalance excitation is assumed in the right end, the respective boundary condition of Equation (3.61) becomes,

\[ V_z(L,t) = \left( s_1 - is_4 \right) U_z(L,t) + \left( s_2 + is_3 \right) \tilde{U}_z(L,t) + \left( p_1 - ip_4 \right) \frac{\partial}{\partial t} U_z(L,t) + \left( p_2 + ip_3 \right) \frac{\partial}{\partial t} \tilde{U}_z(L,t) - m_y r^2 e^{j\omega t} \] (3.64)

![Figure 3.48](image)

**Figure 3.48** Left journal amplitude for the case of anisotropic system with a crack (continuous line: vertical, dashed line: horizontal).

If a crack is introduced in the isotropic system of Figure 3.44, then the frequency response obtained (see Figure 3.45) has double peaks in the 1st, the 3rd and the 5th resonance, as presented in Figure 3.46 (these figures zoom in on the respective resonances). Due to the location of crack at the mid-span, the crack has no effect in the 2nd and the 4th modes since the mid-span in these modes is a nodal point.
3.4.1 Numerical example 2

In the case where the bearing coefficients are not equal to each other in the two main directions, the frequency response displays double peaks, as in the case of the crack, but at much different eigenfrequencies since the system stiffness shift due to bearing anisotropy is much more greater than that due to crack rotation. If the horizontal stiffness is changed to $k_{zz} = 8 \times 10^5$ N m$^{-1}$, the frequency response is changed to that of Figure 3.47. When the crack is introduced, then the frequency response is formed as in Figure 3.48. A detailed look at Figure 3.48 shows that when the crack is introduced, the existing coupling (due to anisotropy) is presented in a different way combined with that due to crack coupling. In Figure 3.49, zooming in on the 3$^{rd}$ resonance shows that when no crack exists, the coupling due to anisotropy is shown by two peaks. Note that this coupling does not need cross-coupling bearing properties in order to be provoked, but can be provoked even with zero cross-coupling bearing properties due to the coupled equations of the shaft motion. When the crack is present, the additional coupling due to the crack arises by introducing two more peaks in the frequency response (see Figure 3.49). Then, there are a total of four peaks in each resonance; two of them are present due to crack breathing coupling and two due to coupled bearing properties and coupled equations of motion.

3.4.2 Conclusions

In this chapter, the problem of coupled bending vibrations in a cracked rotor mounted in resilient bearings was studied. The phenomenon of coupling introduced by a transverse crack...
forces a significant change to the dynamic properties that are already affected by the crack existence. As was shown, the total stiffness in the two main directions experiences a change dependent on the crack local compliance change during rotation. The proposed method in this study offers a continuous model for a cracked rotor-bearing system, including three parameters of coupling: the coupled equations of motion, the coupling due to the breathing crack and the coupling due to cross-coupled bearing coefficients for stiffness and damping.

The observations and the conclusions made from current analysis can be regarded as in following:

1) The coupling affects the response in the time and frequency domains and amplifies the higher harmonics that the crack introduces as components in the vibration.

2) The resilient bearing analysis, including cross-coupling stiffness and damping coefficients, provides evidence that the coupling due to a crack is negligible in respect to the coupling that the bearings introduce.

3) The higher harmonics that are introduced in the vibration spectrum due to the stiffness change caused by the breathing of the crack clearly indicate the existence of a crack since the amplification of 2Xrev harmonic amplitude is increased from the crack cross coupling compliances.

Future work combining the current continuous shaft model with finite bearings could investigate the effect of the coupling due to a breathing crack when the stiffness and damping properties of the bearing are not set for a specific equilibrium position, but instead for a real journal trajectory inside the bearing, since this motion is much more closer to the reality when the system passes through a critical speed.
Chapter 4

Experimental Set Up for the Defected Rotor-Bearing System

4.1 The usage of experimental system

The current experimental set up (see Figure 4.1) was mainly developed during the third year of current work and was designed from initial level in order to fulfill the demand of rotors displacement time histories acquisition so as to provide evidence for those various results that were extracted from simulation. The experimental setup provided various results mainly in the following concepts:

Acquisition of horizontal and vertical displacements in the points of disc and of the shaft in Bearing #1 (see Figures 4.2, 4.3 and 4.4) during system start up or steady state operation.

Effects of rotor cut (see Figure 4.5) in additional harmonic components of acquired time histories during system start up or steady state operation.

The use of horizontal external electromagnetic excitation (transient or steady) as a consideration for cut/crack detection using coupled response measurements.

Effects of worn bearing (see Figures 4.10, 4.11 and 4.12) in additional harmonic components of acquired time histories during system start up or steady state operation.

The use of horizontal external electromagnetic excitation (transient) as a consideration for worn bearing detection using the notification of sub-harmonic component development.

The notification of rotors whirling progress with regard to periodicity or quasi-periodicity of acquired time histories for the cases of intact system, cracked rotor and worn bearing.

4.2 Short description

A brief specification of the entire experimental arrangement parts is given in continue in order to present the current system operational conditions. The current experimental system is designed in order to rotate/vibrate shafts carrying discs in various rotational speeds including those of resonance, and to measure the displacements of the shaft in various points including those of bearings, using displacement sensors/probes. The arrangement can be adjusted in the geometric and constructive demands of various tests.
Chapter 4 – Experimental set up for the defected rotor-bearing system

Figure 4.1 The entire experimental setup. Two PCs with one A/D converter each are used in order to succeed signal acquisition and generation simultaneously.

Figure 4.2 The rotor bearing system with the base and the motor. Presented also bearing housings, sensors and lubricant tubing
since it incorporates moving bearing housings. The machine incorporates continuous lubricating system with adjustable flow and temperature measuring system of the lubricant. The measuring probes are attached around the base of the rotor bearing system without any part of them being in touch with the base in order to avoid disturbances during resonance. The probes are used to measure the displacement in both horizontal and vertical direction, the rotational speed in real time and the Keyphasor®. Also there is the ability of external excitation of the rotor bearing system via two electromagnets located around the shaft at any point.

Figure 4.3 Rotor bearing system with the independently mounted sensor base. The demand for an independently mounted sensor base was unavoidable since the coupling of rotor response and system base response was initially noticed in severe levels during resonance.

4.3 Parts-specifications

The experimental system is formed by the following parts:

Shaft, Fluid film bearings and housings, Lubricating system, Rotating system of the shaft, Electromagnetic exciter, Measuring layout, Mounting base.

The specifications of each part are defined in order mentioned in the following paragraphs.
4.3.1 Shaft

The shaft used in current experiment has to pass at least the first critical speed since this area of rotational speeds is of great interest. Also the radial clearance of the bearing has to be such that the measuring of response inside the bearing to be measurable. In Table 4.1 there is the first critical speed value for uniform shafts of various geometry.

<table>
<thead>
<tr>
<th>D (mm)</th>
<th>L1 (mm)</th>
<th>40</th>
<th>40</th>
<th>40</th>
<th>50</th>
<th>50</th>
<th>50</th>
<th>60</th>
<th>60</th>
<th>70</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>1000</td>
<td>1200</td>
<td>800</td>
<td>1200</td>
<td>1500</td>
<td>800</td>
<td>1200</td>
<td>1000</td>
<td>1500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\omega_{\text{eq}}) (rpm)</td>
<td>7543</td>
<td>4822</td>
<td>3342</td>
<td>9453</td>
<td>4201</td>
<td>2673</td>
<td>11363</td>
<td>5013</td>
<td>1000</td>
<td>1500</td>
<td></td>
</tr>
</tbody>
</table>

*Table 4.1* Undamped (1DOF) critical speed value of uniform shaft of length L and diameter D mounted in hinged ends.

The operational speed range is decided to be up to 5000 RPM and the diameter of shaft is 50mm.
Figure 4.5 The rotors 0.5mm width cut was made with a hand saw proper for steel. The cut location is close to (5mm) disc location in order to benefit the bending moments developed due to disc.

4.3.2 Fluid film bearings and housings

The fluid film bearings have to fulfill the criteria for load carrying capacity and stability in the rotational speed range up to 5000 RPM with the predefined length to diameter ratio (L/D) to be at 0.5. Also the dynamic viscosity of the lubricant has the value of 0.016 Pa Sec in environmental temperature (25°C). The radial clearance is chosen at 100μm. In Table 4.2 the operational parameters of the bearing are presented.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean lubricant Pressure</td>
<td>60 KPa</td>
</tr>
<tr>
<td>Friction coefficient</td>
<td>0.0033</td>
</tr>
<tr>
<td>Friction casualties</td>
<td>6.59 W</td>
</tr>
<tr>
<td>Location of minimum film thickness</td>
<td>65°</td>
</tr>
<tr>
<td>Longitudinal lubricant flow</td>
<td>2.07 lt/min</td>
</tr>
<tr>
<td>Location of maximum pressure</td>
<td>65°</td>
</tr>
<tr>
<td>Input lubricant temperature</td>
<td>35 °C</td>
</tr>
<tr>
<td>Sommerfeld number</td>
<td>0.94</td>
</tr>
<tr>
<td>Friction torque</td>
<td>0.063 Nm</td>
</tr>
<tr>
<td>Minimum fluid film thickness</td>
<td>26μm</td>
</tr>
<tr>
<td>Lubricant flow</td>
<td>4.14 lt/min</td>
</tr>
<tr>
<td>Maximum lubricant pressure</td>
<td>130 KPa</td>
</tr>
<tr>
<td>Location of max. lub. pressure</td>
<td>85°</td>
</tr>
<tr>
<td>Output lubricant temperature</td>
<td>35.3 °C</td>
</tr>
</tbody>
</table>

Table 4.2 Operational/designing parameters of the fluid film bearing
The bearing housings are shown in this chapter in the section of mechanical drawings.

### 4.3.3 Lubricating system

The lubricating system transfers the lubricant from a central tanker to the two bearings with continuous operation. The two bearings are inside a smaller tanker so as to accumulate the lubricant after the bearing exit. The lubricant returns in the central tank using the gravity. The physical properties of the current lubricant are presented in Table 4.3 while in Figure 4.6 the lubricant dynamic viscosity is presented as a function of temperature. Note that the current monotype lubricant offers linear viscosity variation.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kinematical viscosity in 20 °C and in 40 °C</td>
<td>21 mm²/s and 10 mm²/s</td>
</tr>
<tr>
<td>Dynamic viscosity in 20 °C and in 40 °C</td>
<td>0.0183 Pa S and 0.0087 Pa S</td>
</tr>
<tr>
<td>Lubricant density in 40 °C</td>
<td>870 kg/m³</td>
</tr>
</tbody>
</table>

*Table 4.3 Basic physical properties of monotype lubricant SHELL MORLINA 10*

*Figure 4.6 Dynamic viscosity of lubricant SHELL MORLINA 10 as a function of temperature*
Chapter 4 – Experimental set up for the defected rotor-bearing system

The lubricant pump used for the lubricant flow is specified in Table 4.4. The lubricant flows in flexible silicon tubing of internal section diameter Φ4.

<table>
<thead>
<tr>
<th>Operational Voltage</th>
<th>220 AC/380 AC (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotational speed</td>
<td>1450 RPM</td>
</tr>
<tr>
<td>Protection</td>
<td>IP 45</td>
</tr>
<tr>
<td>Current</td>
<td>2/0.9 A</td>
</tr>
<tr>
<td>Power</td>
<td>420 W</td>
</tr>
<tr>
<td>Mounting</td>
<td>Plain roller bearings</td>
</tr>
<tr>
<td>Sound level</td>
<td>73 Db</td>
</tr>
<tr>
<td>Weight</td>
<td>6.6 kg</td>
</tr>
<tr>
<td>Flow supply</td>
<td>2500 lt/h</td>
</tr>
</tbody>
</table>

**Table 4.4 Basic specifications of lubricant pump ROVER POMPE ME B-25**

4.3.4 Rotating system of the shaft

The shaft rotation is succeeded with a coupling of a motor with the following specifications of Figure 4.7.

The motor is controlled from a 3Phase supplier (Inverter) with the specifications of Table 4.5. The inverter (see also Figure 4.11 (a)) is controlled from a logic signal generated from A/D converter (see also Figure 4.11 (b)).

<table>
<thead>
<tr>
<th>Operational voltage - Frequency</th>
<th>3Phase ~ 380-480 V 50-60 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current</td>
<td>3.6 A</td>
</tr>
</tbody>
</table>

**Table 4.5 Specifications of the inverter CONTROL TECHNIQUES – COMMANDER SE 1.5T.**
4.3.5 Electromagnetic Exciter

The electromagnetic exciter is used to harmonically excite the rotational shaft in the lateral horizontal direction. The excitation force is harmonic (sinusoidal) with variable frequency. This kind of excitation has been used in the early past [115,116,119] in order to arise nonlinear resonances due to crack. The exciter is formed by two electromagnets (see Figures 4.8 and 4.9) that each of them apply attractive electromagnetic force with variable frequency (the...
same frequency of the coil current) and of maximum value at almost 10% of total rotor weight (including disc) [122].

Figure 4.8 3D view and operational scheme of electromagnetic exciter

Figure 4.9 The electromagnets arrangement in the horizontal direction provides the external excitation sinusoidal force having the ability of various frequency operation.
Figure 4.10 The electromagnet operation scheme.

Figure 4.11 Arrangement for signal acquisition/generation, motor control, lubricant temperature measurement, signal conditioning, electromagnet supply. In detail: a) 3Phase Inverter used for Motor supply controlled from interface in (b). b) A/D Interface for signal generation. c) 3Phase Inverter for electromagnet current supply controlled from interface in (b). d) Polymeter used for lubricant temperature measurement with a suitable thermocouple. e) Relay switch used for electromagnet current on/off. f) Battery used for Relay operation. g) AC transformer 220V/ 2 X 12V (4A) used for electromagnet supply. h) Signal conditioners for each probe of displacement sensors. i) Signal conditioner supply.
Figure 4.12 Operational scheme of electromagnetic exciter about two different functionalities. The layout in A. offers control in the inverter plugged in the electromagnet and is used when transient electromagnetic excitation is demanded. The layout in B. offers the switching on and off of the electromagnetic excitation that in this case is of steady frequency predefined from the inverter pad.
4.3.6 Measuring layout

The measuring layout is parted from eight (8) inductive sensors that measure displacement using two (2) PCI A/D converters. The total number of channels used in this layout is eight (8), with six (6) of them to be signal inputs and two (2) of them to be signal outputs. In Table C6 there are the basic specifications for the two (2) A/D converters.

<table>
<thead>
<tr>
<th>Type</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) PCI NI-6024 E</td>
<td>Analog. Inputs: (16) SE/ 8 DI 12 Bits 200 KS/s</td>
</tr>
<tr>
<td></td>
<td>Analog. Outputs: (2) 12 Bits 10 KS/s</td>
</tr>
<tr>
<td>(2) PCI NI-6251 E</td>
<td>Analog. Inputs: (16) SE/ 8 DI 16 Bits 1.25 MS/s</td>
</tr>
<tr>
<td></td>
<td>Analog. Outputs: (2) 16 Bits 2.8 MS/s</td>
</tr>
</tbody>
</table>

*Table 4.6 National Instruments A/D converter specifications*

In Table 4.7 there is a description about each channel’s functionality together with the hardware used for each of them. See also Figure 4.13 for further information about measuring layout.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Hardware</th>
<th>Functionality</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0) Input</td>
<td>NI-6024E</td>
<td>Vertical response in Bearing #1</td>
</tr>
<tr>
<td>(1) Input</td>
<td>NI-6024E</td>
<td>Horizontal response in Bearing #1</td>
</tr>
<tr>
<td>(2) Input</td>
<td>NI-6024E</td>
<td>Vertical response in disc location</td>
</tr>
<tr>
<td>(3) Input</td>
<td>NI-6024E</td>
<td>Horizontal response in disc location</td>
</tr>
<tr>
<td>(4) Input</td>
<td>NI-6024E</td>
<td>Keyphasor®</td>
</tr>
<tr>
<td>(5) Input</td>
<td>NI-6024E</td>
<td>Rotational speed</td>
</tr>
<tr>
<td>(0) Output</td>
<td>NI-6251E</td>
<td>Motor frequency control</td>
</tr>
<tr>
<td>(1) Output</td>
<td>NI-6251E</td>
<td>Electromagnet frequency control (chirp)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Electromagnet On-Off (steady)</td>
</tr>
</tbody>
</table>

*Table 4.7 Channels and functionality*
The sensor voltage acquired by the interface is expressed as a function of shaft displacement in Figure 4.14 while the mechanical drawings of the parts of measuring equipment are presented in Figure 4.27 in the section 4.7 with mechanical drawings.
Figure 4.14 Sensor voltage as a function of target displacement. Note that the function is absolutely linear.
4.4 Balancing

The experimental rotor as well as the disc mounted in the rotor includes the defects of rotor permanent bowing and disc unbalance respectively. Both defects are due to asymmetric milling and provoke an unbalance force during rotation that does not matter to be extracted from the system but it is significant to estimate their value so as to be involved in the simulation. For this reason the system is mounted initially in the balancing machine of Figure 4.15 so as to define the unbalance mass. The system is forced to steady state operation of 3000 RPM (a speed far from critical and also in operational range of the system) and the unbalance mass is estimated at \( m_u = 17 \text{gr} \) existing in the radius of \( R_u = 0.019 \) yielding the unbalance of \( u = 0.323 \text{gr m} \).

![Balancing layout used for rotor and disc balancing before the mounting in the fluid film bearings.](image-url)
4.5 Virtual instruments used for operation and data acquisition

In order to achieve the operation of the rotor bearing system and the data acquisition process, some virtual instruments (VI’s) were developed using the compatible with NI DAQ cards software Labview® 7. The operations of the current rotor bearing system demanded for the entire experimental process are presented in following Table 4.9 considering the motor and the electromagnet operational frequency.

<table>
<thead>
<tr>
<th>Case</th>
<th>Motor Frequency</th>
<th>Electromagnet Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>TRANSIENT START UP</td>
<td>NO EXCITATION</td>
</tr>
<tr>
<td>(2)</td>
<td>STEADY</td>
<td>NO EXCITATION</td>
</tr>
<tr>
<td>(3)</td>
<td>STEADY</td>
<td>TRANSIENT START UP (CHIRP)</td>
</tr>
<tr>
<td>(4)</td>
<td>STEADY</td>
<td>STEADY</td>
</tr>
</tbody>
</table>

*Table 4.9 Operational cases of current rotor bearing system*

The front panel together with the block diagram of each VI developed for the current scopes with their functionality descriptions are presented in following Figures.

*Figure 4.16 The current VI generates an output analog voltage 0-10V in order to control the motor inverter and consequently the motor speed. There is the ability of starting/ending speed definition as long as of the rotational acceleration (ramp). If ramp is set equal to zero the motor obtains steady speed equal to the starting speed definition. If starting speed is higher than ending speed then the motor performs transient shut down. The tachymeter is used just for a raw estimation of the speed and does not represent the real rotational speed, that's why a separate input channel is considered so as to estimate real speed in real time using FFT of signal produced in rotational speed ring (see Figure 4.15)*
Figure 4.17 The current VI generates an output analog voltage in order to control the electromagnet inverter and consequently the electromagnetic excitation frequency. The way of operation is similar to this of VI in Figure 4.15.

Figure 4.18 The current VI acquires all the input signals with the desired sampling rate and produce .txt format files with their data. Also the real time rotational frequency of the shaft is estimated by using a real time FFT of signal produced in rotational speed ring (see Figure 4.13).
4.6 Experimental wear progress

A roller bearing is adopted around the shaft near bearing housing #1 as shown in Figure 4.19a; the belt, around the roller bearing, forces the roller bearing and the shaft vertically down with a static load applied from the dynamometer (0-100 kgf) as shown also in Figure 4.19c. The shaft rotation is performed in low speeds such as 200-300 RPM and a force of almost 100 kgf is applied using the dynamometer. The wear process starts without any lubricant flow inside the bearing, having always real time notification of developed temperature around bearing using a thermocouple (see Figure 4.21). The bearing temperature sometimes reaches or overcomes the 90°C with the load and the rotational speed to be the control parameters for the value of temperature, so as not to pass in higher levels that could damage the bearing material.

![Figure 4.19 Arrangement used to apply vertical external force in journal #1 in order to produce wear. a) Plain roller bearing SKF 6010-2RS1. b) Stationary belt used to apply the lateral static load of 175kg from the dynamometer to the rotating shaft. c) Dynamometer 0-200kgf mounted in between belt and system base.](image-url)
Figure 4.20 Journal bearing #1 during wear progress (burs visible)

Figure 4.21 40% worn bearing #1 in housing just after removing the shaft (wear zone visible, with the thermocouple plugged in bearing also visible)
The progress of wear is notified using the sensor that acquires the relative vertical
displacement between bearing housing #1 and journal #1. However the same sensor is used
in acquisition in normal operation. The displacement is acquired until the indication reaches
0.02mm that means 20% of radial clearance additional displacement and the wear progress is
interrupted. Then the external load is removed and the lubricant flow starts with the system
start up to be performed in the same manner as described in previous case of intact system.
Repeating the same procedure the wear progress is continued up to the level of 40%
(0.04mm) of radial clearance and the start up process is also performed. Finally, the system is
disassembled; the bearing is extracted from the housing, and is cleaned in order to notice the
wear zone that follows the Dufrane model described in chapter 1 as it is shown in Figure 4.21
and Figure 4.22. The wear depths that are used in the current survey are $\delta_0 = 0\%$ (not
defected bearings), $\delta_0 = 20\%$ and $\delta_0 = 40\%$. The wear is supposed to exist only in journal
bearing 1 while the journal bearing 2 is intact in order to make damage only in one
experimental bearing and to avoid eventual introduction of motor produced harmonics (near
Bearing #2).
4.7 Mechanical drawings

Figure 4.23 3D view of the entire base with rotor bearing system

Figure 4.24 Side view of the entire base with rotor bearing system
Figure 4.25 Front view of the entire base with rotor bearing system

Figure 4.26 Upper view of the entire base with rotor bearing system
Chapter 4 – Experimental set up for the defected rotor-bearing system

Figure 4.27 Front view and side view of the electromagnets together with shaft

Figure 4.28 Mechanical drawing of probes, proximitor and cables used in the measuring layout
Chapter 5

Nonlinear Effects of Crack in the Dynamic Rotor-Bearing System.

Numerical & Experimental Application

The rotor bearing system composed of a continuous rotor and finite fluid film bearings as it was presented in chapter 1 is used in this chapter to investigate the effect of a crack in system’s dynamic characteristics. Here, the crack is supposed to obtain its form not only due to angle of rotation but also from the magnitudes of bending moments that are developed during rotation and whirling as in section 2.3. It is a fact that the bending moments that act in the cracked section are a function of the rotors displacement as long as rotors loading also.

So, under a little bit different consideration from this presented in chapter 3 the crack can be supposed as breathing, as steadily open, steadily closed or as steadily semi closed or any other different situation that is forced to obtain due to rotation, whirling, loading and every other parameter that can affect bending moments. This different assumption has definitely not to do with any different assumption for local compliances that was made in previous chapters 2 and 3, but establishes a simulation mechanism with which the predefined local compliance is inserted in the model according to which the current crack form is. This assumption was made in order to fulfill the demands of transient formulation of the crack during rotation as it happens when the system starts up and passes through resonance. Also the fact that the experimental progress supposes steadily open crack (because there is cut and not crack) demands a formulation that can yield the appropriate crack form similar to reality.

The main aim of the current chapter is to investigate the crack influence on the system's response and other dynamic characteristics of the frequency and time domain. Experimental setup is presented and comparison between simulation and measurements are made. Also, a cracked rotor bearing system with higher L/D ratio (Length to Diameter ratio) is investigated in parallel (only as a simulation and not experimentally) so as to arise the effects of crack that in such systems are more intense due to higher bending moments.
5.1 The experimental procedure with crack

The current simulation is validated in this section with experimental results from a real experimental system constructed to fulfil the need of comparison with a real rotor bearing system (see chapter 4 for detail). The computational results of time histories are compared with those extracted from current experiment for four different cases of crack/cut depth, 20%, 40%, 60% and 80%. Further, the simulated frequency response during the start up of the system is compared with the corresponding of the real system in order to present the effects of crack/cut in system start up response.

<table>
<thead>
<tr>
<th>Shaft Radius</th>
<th>R = 0.025 m</th>
<th>Material Loss Factor</th>
<th>η = 0.001 (set)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Step Length</td>
<td>L1 = 0.24 m</td>
<td>Bearing Length Lb = 0.025 m</td>
<td></td>
</tr>
<tr>
<td>2nd Step Length</td>
<td>L2 = 0.414 m</td>
<td>Bearing Radial Clearance Cr = 100 μm</td>
<td></td>
</tr>
<tr>
<td>Oil Viscosity</td>
<td>μ = 0.006 Pa s</td>
<td>Unbalance 0.323 gr m</td>
<td></td>
</tr>
<tr>
<td>Young Modulus</td>
<td>E = 2.068 GPa</td>
<td>Shaft Density ρ = 7832 kg/m³</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1 Geometric and physical properties of the experimental rotor bearing system.

Note that the extracted time histories are acquired with a sample rate of 5000 samples/s. Also a denoising was performed using wavelet decomposition and finally, an averaging was used so as to smooth the plotted time signal. The start up is performed in the following manner: at first the system is set in a steady state rotation of \( \Omega = 50\text{rad/s} \) and the lubricant is heated to the temperature of 45°C so as to achieve the desired viscosity of almost 0.006 Pa s. Then the rotor is forced to perform a transient operation of steady rotational acceleration of 12RPM/s or 1.25 rad/s² and the acquisition is triggered to start. In the time of almost 208s the maximum speed of 300 rad/s is achieved and the acquisition is disabled. The exactly similar way of starting up is used in simulation also so as to avoid different transient phenomena.

The cut is made with a 0.5mm saw (see Figure 5.1b) and obtains gradually two different depths of 10mm (40% of radius) and 15mm (60% of radius). It is understandable that the cut remains open during rotation and cannot receive compression load. However, as it is explained in continue and in detail and as it has been comprehensively explained in chapters 2 and 3, the cut can introduce time variable local stiffness coefficients while rotation and additionally can introduce the coupling phenomenon due to local section asymmetry. Thus the experimental cut can be used in order to compare results with the corresponding simulation
results that incorporate crack. The results of the effects of cut and crack are commented in
detail in following sections.

Figure 5.1 a) View and c) Mechanical drawing of the experimental set up of the current cracked rotor fluid film
bearing system. b) photo of 0.5mm cut in the rotor in order to incorporate coupling phenomena due to local
compliance. c) Mechanical drawing of current rotor bearing system.
5.2 Results for time response

Considering the experimental rotor bearing system of Figure 5.1 with the properties of Table 5.1, measured time histories of disc center during start up are acquired, denoised by averaging, and plotted in Figure 5.3 for vertical direction while in Figure 5.2 the corresponding simulated time histories are presented. The cut depth becomes 40% and 60% yielding the time histories of Figures 5.2 and 5.3.

The disc center response includes the symptoms of crack/cut intensively but in real machines the use of journal response (bearing measurements) is of great importance. The journal response of the cracked system is analyzed in the last chapter (chapter 7) because its use has to do with the consideration of detection. The horizontal response is presented in continue in Figures 5.4 and 5.5. From the measured and evaluated time histories, a clear symptom introduced from the crack defect cannot be extracted, but in continue the time
histories are analyzed using Short Time Fourier Transform (STFT) in order to reveal symptoms in time and frequency domain. For the moment, the presentation of simulation time histories reveals a response progress similar to this of real system implying an adaptability of simulation code in system properties (especially bearing properties).

![Simulation Time Histories](image1.png)

Figure 5.4 Simulated time histories of disc centre horizontal response during start up for a) crack depth 0%, b) crack depth 40% and c) crack depth 60%

![Experimental Time Histories](image2.png)

Figure 5.5 Experimental time histories of disc centre horizontal response during start up for a) crack depth 0%, b) crack depth 40% and c) crack depth 60%

The fact that the mathematical model can be adapted to the behaviour of the real one in various speeds is the result of nonlinear fluid film consideration and the current combination of continuous rotor with journal bearings. The current simulation converges with the measured rotor response amplitude in speeds even near critical speed, where different theories yield results even out of bearing radial clearance. Also the current convergence of computational and experimental critical speed, as it will be shown in continue, validates that the current model can adapt the system properties in an efficient way that means good definition of bearing stiffness and damping properties as a function of system’s response. Different theories assuming linear bearings or even nonlinear bearings including the usage of nonlinear stiffness and damping coefficients cannot leave out the usage of equilibrium position, instead.
of current theory that does not assume equilibrium position as it happens in many cases of rotational speeds.

Figure 5.6 Journal 1 trajectories of experimental rotor bearing system and simulation for various rotational speeds during start up. a) 50 rad/s, b) 100 rad/s, c) 150 rad/s, d) 200 rad/s, e) 250 rad/s, f) 300 rad/s.

Figure 5.7 Disk center trajectories of experimental rotor bearing system and simulation for various rotational speeds during start up. a) 50 rad/s, b) 100 rad/s, c) 150 rad/s, d) 200 rad/s, e) 250 rad/s, f) 300 rad/s. Circle stands for bearing radial clearance.
In Figures 5.6 and 5.7, journal and rotor orbits are presented as results of algorithm evaluation as well as experimental measurements, for various rotational speeds. As it is shown in Figures 5.6 and 5.7 there is an adaptability of the evaluation code in linear and nonlinear behavior that the system presents in rotational speeds higher than first critical speed, as shown in the experimental measurements. Additionally, there is a convergence in the whirling amplitude between evaluation and measurements. However, the simulated and measured time responses are not similar to each other due to various parameters that are not or cannot be incorporated in the simulation.

5.2.1 Time histories of cracked system with different (larger) slenderness ratio L/D (simulation only)

It is a fact that rotor systems with larger slenderness ratio L/D can be more proper to present the symptoms of the crack due to the ability of developing higher bending moments but on the other hand are not proper to be used in real machines (either experimental) because the amplification factor that characterizes such systems is higher and the start up through resonance can not be always safe. In correspondence to the previous results of time histories of experimental and simulation system of slenderness ratio L/D=13 (see Table 5.1) an alternative rotor bearing system is used for another simulation with a slenderness ratio of L/D= 40 (see Table 5.2). Such a system cannot be built for experimental procedure due to the reasons pronounced but its simulation provides in a very clear way the symptoms of crack in time response.

The case of the continuous cracked rotor with high L/D ratio mounted in finite bearings is investigated in this paragraph, using the crack model of Section 2.3. The current analysis (with crack) requires also the same 2STEP rotor because the crack is located at the same point (with the disk); this is due to the fact that both crack and disk can be imported only with boundary conditions. The crack depth in the following analysis is variable with \( \alpha = 40,60\% \). All the geometrical and physical properties as long as the algorithm parameters remain the same as in Table 5.2. It is known that a crack affects dynamic characteristics such as critical speeds and frequency response function, and also affects the time domain response by introducing additional harmonics. The effects in time domain are the target of this section of high L/D ratio system.

A similar to Figure 5.1c rotor bearing system, with physical and geometric properties given in Table 5.2 is used in this paragraph. The simulative start-up of the system is performed from the initial rotational speed \( \Omega = 25\text{rad/s} \) up to the maximum \( \Omega = 100\text{rad/s} \) with an acceleration of \( \dot{\Omega} = 4\text{rad/s}^2 \), while the sampling frequency is \( 1/\Delta t = 800\text{Sam.}/\text{s} \). Note that the sampling
frequency, a significant parameter, is a result of various tests in order to make the algorithm computable. A time step of $\Delta t = 0.00125 \, s$ is used in all evaluations. This value complies with the Nyquist theorem for minimum sampling rate, offers stability in the finite difference approach to dynamic variables, and makes the solutions computable for various choices of the geometrical and physical properties of the system. Also, another parameter that is determined for the convergence of the evaluation is the perturbation $\Delta q$ needed for calculation of the Jacobean matrix in Equation (1.124). This parameter is set to $\Delta q = 1E-12$.

<table>
<thead>
<tr>
<th>Shaft Radius</th>
<th>$R = 0.025 , m$</th>
<th>Material Loss Factor</th>
<th>$\eta = 0.001$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Step Length</td>
<td>$L_1 = 1 , m$</td>
<td>Bearing Length</td>
<td>$L_b = 0.025 , m$</td>
</tr>
<tr>
<td>2nd Step Length</td>
<td>$L_2 = 1 , m$</td>
<td>Bearing Radial Clearance</td>
<td>$C_r = 100 , \mu m$</td>
</tr>
<tr>
<td>Disk Radius</td>
<td>$R_d = 0.19 , m$</td>
<td>External Load</td>
<td>$E_F = (9.81 , m d) , N_t$</td>
</tr>
<tr>
<td>Disk Density</td>
<td>$\rho = 7832 , kg/m^3$</td>
<td>Rotor Static Bowing</td>
<td>$e_b = 0.00025 , m$</td>
</tr>
<tr>
<td>Disk Width</td>
<td>$L_d = 0.022 , m$</td>
<td>Unbalance Force</td>
<td>$F_u = (e_b , m_d , \Omega^2) , N_t$</td>
</tr>
<tr>
<td>Oil Viscosity</td>
<td>$\mu = 0.002 , Pa , s$</td>
<td>Disk Mass</td>
<td>$m_d = 19.54 , kg$</td>
</tr>
<tr>
<td>Young Modulus</td>
<td>$E = 2.068 , GPa$</td>
<td>Shaft Density</td>
<td>$\rho = 7832 , kg/m^3$</td>
</tr>
</tbody>
</table>

Table 5.2 Geometric and physical properties of the rotor bearing system with high L/D ratio

However, the material loss factor is set arbitrarily low to just "cut" the infinite response so as to make the startup computable. In this work, the variable loss factor is not included because the internal damping is treated as a tool in order to avoid the infinite response that cannot be damped by the bearing damping coefficients.

The system response during startup is plotted in Figures 5.8 to 5.11 for the cases of crack depth pronounced.
Figure 5.8 Time history during start up for a) Journal 1, b) Mid-span response for 20% cracked rotor.

Figure 5.9 Time history during start up for a) Journal 1, b) Mid-span response for 40% cracked rotor.

Figure 5.10 Time history during start up for a) Journal 1, b) Mid-span response for 60% cracked rotor.

Figure 5.11 Time history during startup for a) Journal 1, b) Mid-span response for 80% cracked rotor.
Effects in the time domain response are investigated in this example using the current model and are aimed at a possible differentiation by virtue of the fact that the crack can affect not only the shaft but also the journal response. The current crack model, as described in Section 2.3, introduces an additional coupling between the vibrations in the two main directions, the vertical and the horizontal. The crack is assumed to open and close with rotation, and it is known that the breathing behavior introduces higher harmonics in the time response of the rotor, as well as an additional resonance at a rotational speed of one-half of critical ($\Omega_\text{cr}/2$) and sometimes even in one-third of critical ($\Omega_\text{cr}/3$). These results were also validated in the current analysis. Initially, the Journal #1 and Mid-span response for all cases of crack depth are computed as in Figures 5.8 to 5.11. The fact that additional resonances appear in rotational speeds of $\Omega_\text{cr}/2$ and $\Omega_\text{cr}/3$ is a success due to the expression of the response (see chapter 1) avoiding the harmonic components of $\cos(\Omega t)$, $\cos(2\Omega t)$, $\cos(3\Omega t)$ etc, as it is traditionally used in literature. By introducing the components of $\cos(\Omega t)$, $\cos(2\Omega t)$, $\cos(3\Omega t)$ etc, in the time solution of differential equations of motion obliges, in some way, the development of 2XRev and 3XRev components in the response as in [214]. In current analysis the time solution is not declared (see Equations (1.38)-(1.45)) and this means that every additional component is developed due to every reason of nonlinearity without being forced to obtain a frequency of a pre-declared value such as $2\Omega$ or $3\Omega$ etc but a value depended from the progression of the solution. For instance, if the model contains a crack additional components will be developed of any integer multiple frequency with the 2XRev to be amplified.

As shown in Figures 5.10 to 5.9, the time response during start up is clearly affected only when the crack depth is 40% or higher, with the additional resonance of $\Omega_\text{cr}/2$ clearly developed in Mid-Span when the crack depth is about 60% or higher, and the additional resonance of $\Omega_\text{cr}/3$ lightly developed in crack depths above 60%. These notations are made regarding the sum of all components that make up the response, but using the Short Time Fourier Transform, the progression of each component can be noted during startup as it will be shown in another section of this chapter. Also, in continue, the frequency response estimation for Journal #1 and Mid-span should be observed in order to note the slight shift in critical speed and the additional resonance development due to the crack.
5.3 Inspection of periodicity – quasiperiodicity

Poincaré maps are used in this analysis to identify the inclusion of aperiodic or chaotic motion in the dynamic system flow. A point on the Poincaré section can be referred as the point of the trajectory (Z-Y plane) in which the system will return after a time amount of $T$, where $T$ is the period of the driving excitation force. The projection of a Poincaré section on the $Z(nT) - Y(nT)$ plane is referred as the Poincaré map of the system.

If the motion of the system is periodic and synchronous with the driving period $T$ then the Poincaré map will be consisted from only one point and this is obvious since the rotor will pass through the same point and with the same velocity every time amount $T$. This means that even the projection of the Poincaré section is on the $Z(nT) - Y(nT)$, the Poincaré map of the system is consisted from only one point. For the case of periodic motion with period $2T$ or generally $nT$ ($n \in \mathbb{Z}^+$) there are $n$ discrete points in the Poincaré map. Also there is a situation in which the points of the Poincaré map consist a closed trajectory or a non fractal geometric shape and this is explained as a quasi-periodic motion instead of the case of a geometrically fractal structure of the Poincaré map that betrays a chaotic motion-attractor.

![Figure 5.12 Disk centre experimental time history poincaré maps for cut depth 0% in a) 100 rad/s, b) 200 rad/s, c) 300 rad/s.](image)

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The experimental time histories of steady state operation of the cracked system reveal periodic oscillations of period $T$ and period $2T$ as the rotational speed increases further from critical. The ability of algorithm to produce oscillations with periodicity or aperiodicity was remarked in chapter 1, thus in this section experimental results are only presented so as to present the real system response character.

It is a fact that the algorithm can produce aperiodic response in different speeds from the real system, and this fact has to do with various parameters of the real system that are not incorporated in simulation. However the defect of crack affects the character of periodicity as shown in Figures 5.12 to 5.14. The oscillations are periodic in every speed of 100, 200, and 300 rad/s and synchronous to driving period when no crack is introduced (see Figure 5.12), while in Figure 5.14 it is shown that 2T period is noticed in oscillations of 60% cracked system in rotational speeds of 100 rad/s (almost half of critical) and 300 rad/s. However, the use of Poincaré maps as a tool for crack detection cannot be assumed at the moment since various parameters can affect the whirling periodicity.
5.5 Results for frequency response

The response amplitude of time response evaluated and measured in section 5.2 is extracted and plotted as a function of rotational speed in Figure 5.15. As it will be commented in detail in next section 5.5.1, the frequency response of the cracked system of low ratio L/D is nominally shifted in comparison of the intact one, and this due to the fact that the bending moment development is not such high so as to effect the crack symptoms. For this reason, the critical speed shift is nominal as shown in Figure 5.16.

![Graphs showing frequency response for various components of a dynamic rotor-bear system](image)

**Figure 5.15** a) Journal 1 horizontal amplitude, b) Journal 1 vertical amplitude, c) Disk center horizontal amplitude, d) Disk center vertical amplitude.
The adaptability of the algorithm in the magnitude of the response amplitude can be presented in Figures 5.15 and 5.16. It is a benefit that the response amplitude yield by simulation near critical speed is close to this of experimental measurement. This fact is due to the adaptability of bearing properties in the proper values of stiffness and damping coefficients.

5.5.1 Frequency response of cracked system with different (larger) slenderness ratio L/D (simulation only)

The response amplitude of time response evaluated in section 5.2.1 is extracted and plotted as a function of rotational speed in Figure 5.20.
Chapter 5 – Nonlinear effects of crack in the dynamic rotor-bearing system. Num. & exper. application

![Figure 5.17 Frequency responses in a) Journal 1 and b) Mid-span for variable crack depths.](image)

It is a fact that the critical speed of the system decreases as the crack depth increases as it is shown also in Table 5.3 and the decrement is more intense comparing with the results of experimental system with low L/D ratio. The critical speed shift due to crack is judged insufficient to provide information about crack detection since this shift is nominal even in high cut/crack depths. Additionally the critical speed shift of such a system can be affected from other parameters such as lubricant viscosity.

<table>
<thead>
<tr>
<th>Crack depth</th>
<th>Critical speed, rad/s (experiment/simulation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>Low L/D ratio (13) 216/210</td>
</tr>
<tr>
<td>40%</td>
<td>High L/D ratio (40) -75</td>
</tr>
<tr>
<td>60%</td>
<td>-73</td>
</tr>
<tr>
<td></td>
<td>-70</td>
</tr>
</tbody>
</table>

Table 5.3 Critical speed estimation for the two cases of high and low L/D ratio concerning Figures 5.13 and 5.14.
5.6 Results for time – frequency decomposition using CWT

Using the Short Time Fourier Transform (STFT), the time histories presented section 5.2 are analyzed in time and frequency domain in order to inspect the additional, in the synchronous response, harmonic components during the entire time domain of starting up. In Figures 5.18 to 5.20 the specgrams of vertical response time histories during start up in experimental and simulation process are presented for the crack depths of 0%, 40%, 60% proving that the additional harmonic development due to crack is noticed even in low speeds.

Figure 5.18 Magnitude of Short Time Fourier Transform (specgram) for time histories of vertical response in disc center for the case of 0% cut depth. a) experiment, b) simulation.

Figure 5.19 Magnitude of Short Time Fourier Transform (specgram) for time histories of vertical response in disc center for the case of 40% cut depth. a) experiment, b) simulation.
The simulation time histories as long as the experimental ones provide the development of hyper harmonics with frequencies 2XRev, 3XRev, 4XRev, and higher even in time moments that the rotational speed is low (in 20s). In the specgrams of simulated time histories the aliasing phenomenon appears due to low sampling rate of 200 samples/s (dt = 0.005s) with this value to be a demand for feasible code evaluation. However the experimental time histories are sampled with sampling rate 5000samples/s and do not present aliasing in such a magnitude as in simulation. Also, the low sampling rate of evaluation do not provide good frequency estimation, with the synchronous frequency to reach 500rad/s in time moment of 180s, but since the area of interest remains in low frequencies the frequency localization is not a matter of significance.

5.6.1 Time Frequency analysis of cracked system response with different (larger) slenderness ratio L/D (simulation only)

The fact that the fluid film forces are strongly nonlinear affects the rotor motion by introducing asynchronous harmonics into the driving force frequency. This fact can be observed in the cases where the fluid film forces can be characterized as nonlinear, and is dependent on the rotational speed and unbalance force. Using the current algorithm, the nonlinearity in fluid film forces is a property that is adopted in the system in the case where the rotor journals are forced to execute motions far from equilibrium positions. For this reason, time frequency analysis using Short Time Fourier Transform (STFT) is performed in order to investigate the development of additional harmonics in Journal 1 (the left journal) and the mid-span time response. The technique of STFT is preferred in current analysis since the analyzed time histories...
histories does not contain sudden appeared components so as a well time localized
decomposition like wavelet transform or Wigner-Ville decomposition to be needed. However
Wigner-Ville Decomposition as long as Wavelet Transform were used (without to be
presented here) in order to inspect differences, without yielding a useful result. In current
application a windowed decomposition like STFT is judged sufficient for the current time
histories and additionally STFT appears to be well localized in frequency and time domain in
those time histories.

In following the Short Time Fourier Transform of the start-up time histories of section 5.2.1, is
computed and plotted in Figures 5.21, 5.22, and 5.23 for crack depths \( \pi \) of 0\%, 40\%, and
60\%, respectively. As it is shown in Figure 5.21 (intact system), the Journal #1 time series
does contain harmonics different from those of synchronous and 2Xrev when the rotational
speed is about 70 rad/s or lower, because at this speed there is a high journal amplitude
response that could produce strong nonlinearity in the fluid film forces. Note that the current
rotational speed of 70 rad/s is near the critical speed. It is clear that when the rotational speed
is increased further, i.e., 90 rad/s (time: 16s), the time series in rotor Mid-span and in Journal
1 contain higher harmonics introduced by the fluid film non-linear forces (see Figure 5.23), but
their magnitude begins to decrease when the systems passes the resonance. At rotational
speeds around critical (\( \Omega = 75 \) rad/sec, time: 12.5s), all harmonics increase in amplitude in
Journal 1, but at Mid-span there is only an increase in the 2Xrev harmonic.

Having a detailed look in Figure 5.22 and comparing with the respective result for the
uncracked case (Figure 5.21), it can be seen that even for a crack depth of 40\%, an
increment in 2Xrev harmonic amplitude is observed in Mid-span time history at about the time
that the system passes through the \( \Omega_{cr} / 2 \) rotational speed. Also the time moment passing
through critical speed \( \Omega_{cr} \), the 2Xrev harmonic increases more comparing with the case without
crack. An increment may be present, even for lower crack depths, but such an observation
required further signal processing techniques usually performed by subtracting the “non-
defected” signal from the “defected” one, but such an analysis is beyond the aim of this
chapter since here the concept is not the detection.

As the crack depth increases to 60\% (see Figure 5.23), a further increment in 2Xrev harmonic
amplitude is observed in Journal #1 and Mid-span time histories at about the time when the
speed passes through the \( \Omega_{cr} / 2 \); also, a slight increment in the 3Xrev harmonic begins in
Journal #1 and Mid-span time histories at times below \( \Omega_{cr} / 2 \) and near \( \Omega_{cr} / 3 \), with poor time
localization due to the current Time-Frequency Decomposition.
Figure 5.21 Magnitude of Short Time Fourier Transform (specgram) for time histories of vertical response in
a) Journal 1, b) Mid-span, for the case of 0% cracked rotor.

Figure 5.22 Magnitude of Short Time Fourier Transform (specgram) for time histories of vertical response in
a) Journal 1, b) Mid-span, for the case of 40% cracked rotor.

Figure 5.23 Magnitude of Short Time Fourier Transform (specgram) for time histories of vertical response in
a) Journal 1, b) Mid-span, for the case of 60% cracked rotor.
5.7 Results for coherency estimation

In order to see the effect of the crack in the property of system non-linearity, the coherency estimation is performed between the two signals of Unbalance in Midspan and Journal 1-response for the cases of steady rotational speeds. Actually, a cut of relative depth of 40% to shaft radius is made in the experimental shaft. The cut is of 0.5 mm width and thus remains open during rotation. In current work, this cut aims to produce additional nonlinear characteristics in the systems response similar to those that a breathing crack produces (in simulation) since both defects (crack and cut) introduce variable local stiffness characteristics. The main aim is to get an indication of the strength of nonlinearity that variable stiffness due to cut introduces in the system in comparison to the nonlinearity introduced by the fluid film bearings. Note that the unbalance force is defined as in the end section 2.2.1 as

\[ F_u = m_u \left( R_e + v_y + \sqrt{Y_{1,2} (L_1, \omega t)^2 + Z_{1,2} (L_1, \omega t)^2} \right) \Omega^2 \]

and thus the unbalance force becomes a function of systems response, since the elastic deflection of the shaft has an effect in unbalance radius of rotation.

This comparison is made using coherency estimation of two time histories: The journal response as an output, and the unbalance force as an input. For the inspection of bearing nonlinearity the system response is acquired in three different steady rotational speeds of \( \Omega = 100, 200, 300 \text{rad/s} \) so as to produce three different unbalance force magnitudes. Under the different unbalance force magnitude the journal whirl amplitude would be of different value and would produce different nonlinearity.

The coherency estimation for that case is presented in Figure 5.24 with the dashed line (intact system since cut depth is zero). It is shown that the synchronous frequency component coherency is of a value 0.95 in the rotational speed of 100 rad/s (Figure 5.24a), of a value 0.65 in the rotational speed of 200 rad/s (Figure 5.24b), and of a value of 0.85 in the rotational speed of 300 rad/s (Figure 5.24c). So the coherency value can be said to obtain a range from 0.85 to 0.95 at three different speeds in the entire operational speed range of current speed. In contrast, the cut introduces much lower coherency values in all speeds of inspection. As shown in Figure 5.24a, the cut provokes a coherency fall in synchronous component from 0.95 to 0.58 in the rotational speed of 100 rad/s, from 0.65 to 0.07 in the rotational speed of 200 rad/s (Figure 5.24b), and from 0.85 to 0.3 in the rotational speed of 300 rad/s (Figure 22c). The same variation stands for the multiple higher harmonics 2Xrev and 3Xrev presented in the current frequency range of the plots.
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Figure 5.24 Coherency estimation of journal 1 response and disk unbalance force for the rotational speeds of a) 100 rad/s, b) 200 rad/s, c) 300 rad/s.

It is a fact that both reasons of nonlinearity introduction are coherent to each other since the fluid film bearing forces are depended to the response of the system. There can not be an “isolation” of the nonlinear response due to the crack from this due to the nonlinear fluid film forces and for this reason the coherency estimation cannot be distinguished in “crack produced” and “bearing produced” when a crack is present in the system.

5.8 Conclusions for cracked model

The main aim of this chapter was to present a different model for the construction of a rotor bearing system combining an internally damped continuous Rayleigh shaft with finite fluid film bearings in a such way that no bearing coefficients are used (as in most research up to now). This model's ultimate goal is to achieve high accuracy, even with unpredictable rotor-journal motions in any location in the radial clearance area, and also to investigate how this model yields the already known from the literature results in the effect that a breathing crack with coupling compliance has on time-frequency domain dynamic properties. The internal damping is consisted by the internal hysteretic damping of rotor and of the internal viscous damping from the fluid film bearings. The current investigation steps away from the concept of identification because the main focus is in developing a better approach to the rotor bearing dynamics; the current work can be further developed in future studies for the purpose of crack identification. Since the current system can be considered as a dynamic system, there was a need to investigate the progress of the response by decomposing the time histories in their components. Additionally an experimental validation with an intact real rotor-fluid film system is made with a very good convergence in basic magnitudes such as the frequency response and time response.
From the current analysis, some useful conclusions can be made:

Very good convergence between the current developed simulation and the intact real rotor-bearing system is noticed. The simulation converges to the real system in the magnitudes of time response and frequency response with the additional harmonic components of the time domain to be developed in a similar way both in simulation and experiment.

Comparison with similar simulation of the literature proves that the current simulation can provide better the impedance coefficients of the fluid film bearing in the time that the system is in resonance. Many of previous works keep the same stiffness and damping coefficients before and after resonance without to provide the current simulation accuracy.

Even without defects, the nonlinearity in fluid film forces causes the development of higher harmonics, of frequency 2xrev or 3xrev and higher, in rotational speeds near the critical speed as it is happens also in the respective real system.

The rotor is forced to execute motions in the major area below the radial clearance when the rotational speeds approach the critical value; this fact is due to the combination of the developed shearing forces and the fluid film impedance. The experimental procedure proves similar progression of the response.

When the crack defect is introduced, all harmonics are amplified due to the breathing behavior, even those harmonics that are not appeared in the intact system. An interesting observation is that additional resonances at $\frac{\Omega_{cr}}{2}$ and $\frac{3\Omega_{cr}}{3}$ are developed without to be simulated to do so (see chapter 1). To explain further, the non-linear resonances in other models of cracked rotors are presumable since the equations of motion contain the $2\Omega_r$ and sometimes the $3\Omega_r$ components of the response, but in the current analysis, the progression in time is not “forced” from a harmonic function such as $\cos(2\Omega_r)$ or $\cos(3\Omega_r)$, and is left undefined so as to be a result of the system progression.

The coherency of the input of unbalance force and the output of response is strongly depended from the rotational speed and the presence of the crack defect. The coherency between unbalance and journal response is kept lower than this of unbalance force and mid-span response in most cases. Similar observations of coherency magnitude are presented in the real system.

The current dynamic system is characterized by adaptability in linear and nonlinear behavior depending on the fluid film impedance and defect presence, without any adjustment in bearing properties. This adaptability arises from the fact that the method the bearings are
introduced in the shaft can conform to any situation in which the linear bearing consideration is judged insufficient.

It has to be noticed that the use of finite fluid film bearings make the current work different enough from corresponding works using usual ball bearings since the stiffness and damping properties of usual ball bearings are much different from those of fluid film bearings. Some results due to crack presence can occur in rotor systems on ball bearings but since the use of ball bearings is limited enough in large rotating machinery (mainly due to week damping properties and other disadvantages), the current work prefers to provide the combination of the cracked rotor and the complex enough element of fluid film bearing that in operating speeds near critical, yields much more different properties of damping in comparison to ball bearings. As an additional result and under the consideration of literature, there are not main differences in the effects that a crack introduces in rotor-fluid film and in rotor-ball bearing systems, but taking into account the damping properties of the fluid film bearing, the extended usage, and the various additional effects that introduce in rotor bearing operation arise a consideration about how the fluid film bearings interact with the crack effects in the same nonlinear system.

Future work includes the investigation of aperiodic behavior of rotor bearing systems during the start up since the current simulation can provide periodic and quasi-periodic motions of the rotor with the periodicity and non-periodicity to be exchanged during the spinning speed variation as it widely has been investigated in the literature.
Chapter 6

Nonlinear Effects of Bearing Wear in the Dynamic Rotor-bearing System

Numerical & Experimental Application

The rotor bearing system of chapter 5 is used also in this chapter to investigate the effect of worn journal bearings in system’s dynamic characteristics. The main aim of the current chapter is to investigate the wear influence on the system’s response and other dynamic characteristics of the frequency and time domain. Experimental setup is presented and comparison between simulation and measurements are made.

6.1 The experimental set up with worn bearings

In this chapter the experimental setup for the worn system will be described (see also entire Appendix C in detail) and the experimental results will be presented extensively and in comparison with the respective results from simulation code. Consider the rotor bearing system of Figure 6.1 with the physical and geometric properties given in Table 6.1.

The current geometry has been chosen so as to correspond to a continuous shaft, making the effect of slenderness ratio at least present, according to [4]. To explain further, the slenderness ratio of system in Figure 6.1 is $R/2L = 0.02$ (length to diameter aspect ratio $L/D=13$) and according to [4], the slenderness ratio value in which critical speed calculation theory has important role in critical speed value is around $R/2L = 0.01$. According to current geometry the system can be characterized as a continuous shaft rather than Jeffcott rotor.

<table>
<thead>
<tr>
<th>Shaft Radius</th>
<th>$R = 0.025$</th>
<th>Material Loss Factor</th>
<th>$\eta = 0.001$ (set)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Step Length</td>
<td>$L_1 = 0.240m$</td>
<td>Bearing Length</td>
<td>$L_s = 0.025m$</td>
</tr>
<tr>
<td>2nd Step Length</td>
<td>$L_2 = 0.414m$</td>
<td>Bearing Radial Clearance</td>
<td>$c_r = 100 \mu m$</td>
</tr>
<tr>
<td>Disk Radius</td>
<td>$R_d = 0.19m$</td>
<td>External Load</td>
<td>$E_F = W_d \times N_t$</td>
</tr>
<tr>
<td>Rotor Static bowing</td>
<td>$e_b \approx 0.0015m$</td>
<td>Unbalance mass</td>
<td>$m_u \approx 0.017kg$</td>
</tr>
<tr>
<td>Shaft/Disk Density</td>
<td>$\rho = 7832kg / m^3$</td>
<td>Oil Viscosity</td>
<td>$\mu \approx 0.013Pa S$</td>
</tr>
<tr>
<td>Disk Width</td>
<td>$L_d = 0.022m$</td>
<td>Young Modulus</td>
<td>$E = 2.068GPa$</td>
</tr>
</tbody>
</table>

*Table 6.1 Geometric and physical properties of the current experimental rotor bearing system*
Additionally, the current geometrical and physical properties of Table 6.1 correspond to a real experimental rotor bearing system; see Figure 6.1 which has been set up in order to fulfil the comparison of simulated and measured results for time histories and critical speed. The disk is located in 1/3 of the entire shaft length in order to produce gyroscopic moments during the passage of 1st critical speed. Further, the experimental rotor bearing system is designed so as to pass API code demands, as it will be shown in continue.

Explaining now the experimental procedure, two time histories are extracted during the evaluation: The vertical and horizontal response of Journal 1. It is implied that the responses
in disc location are also analyzed but, since the results are similar, we prefer not to present it (for space economy reasons). The extracted time histories are acquired with a sample rate of 5000 samples/s. Also a denoising was performed using wavelet decomposition and finally, an averaging was used so as to smooth the plotted time signal. The start up is performed in the following manner: at first the system is set in a steady state rotation of $\Omega = 50 \text{rad/s}$ and the lubricant is heated to the temperature of near $35 \, ^\circ \text{C}$ so as to achieve the desired viscosity of almost $0.013 \text{Pa s}$ (Lubricant type: Shell Morlina ISO 10). Then the rotor is forced to perform a transient operation of steady rotational acceleration of $12 \text{RPM/s}$ or $1.256 \text{rad/s}^2$ and the acquisition is triggered to start. In the time of almost $210 \text{s}$ the maximum speed of $300 \text{rad/s}$ is achieved and the acquisition is disabled. The exactly similar way of starting up is also used in simulation code in order to avoid different transient phenomena.

After the acquisition of intact system start up, the procedure of bearing wear is performed. A roller bearing is adopted around the shaft near bearing housing #1 as shown in Figure 6.2a with the belt around it to force the roller bearing and the shaft vertically down with a static load applied from the dynamometer (0-100 kgf) shown also in Figure 6.2a. The shaft rotation is performed in low speeds such as 200-300 RPM and a force of almost 100 kgf is applied using the dynamometer. The wear process starts without any lubricant flow inside the bearing, having always real time notification of developed temperature around bearing using a thermocouple (see Figure 6.2c). The bearing temperature sometimes reaches or overcomes the $90 \, ^\circ \text{C}$ with the load and the rotational speed to be the control parameters for the value of it, so as not to pass in higher levels that could damage the bearing material.
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Figure 6.2 a) Arrangement used to apply vertical external force in journal #2 in order to produce wear. b) Journal bearing #2 during wear progress (burs visible). c) 40% worn bearing #2 in housing just after removing the shaft (wear zone visible, with the thermocouple plugged in bearing also visible). d) 40% worn bearing after disassembly and cleaning (wear zone visible also here).

The progress of wear is notified using the sensor that acquires the relative vertical displacement between bearing housing #1 and journal #1. However, the same sensor is used in acquisition in normal operation. The displacement is acquired until the indication reaches 0.02mm that means 20% of radial clearance additional displacement and the wear progress is interrupted. Then the external load is removed and the lubricant flow starts with the system start up to be performed in the same manner as described in previous case of intact system. Repeating the same procedure the wear progress is continued up to the level of 40% (0.04mm) of radial clearance and the start up process is also performed. Finally, the system is disassembled; the bearing is extracted from the housing, and is cleaned in order to notice the wear zone that follows the Dufrane model described in chapter 4 as it is shown in Figure 6.2c and Figure 6.2d. The wear depths that are used in the current survey are $\delta_0 = 0\%$ (not defected bearings), $\delta_0 = 20\%$ and $\delta_0 = 40\%$. The wear is supposed to exist only in journal bearing 1 while the journal bearing 2 is intact in order to perform damage only in one experimental bearing and to avoid eventual introduction of motor produced harmonics (near Bearing #2).

In continue various experimental results about time histories, frequency response, and time-frequency analyses are presented for these three cases of wear (0%, 20%, and 40%) together with the respective simulation code results.
6.2 Results for time response

In this section the acquired and simulated time histories of journal #1 are presented and processed so as to obtain rotor orbits for both experiment and simulation.

In Figure 6.3 the measured vertical and horizontal time histories during start up are presented for the variable wear depth, while in Figure 6 the corresponding simulated time histories are plotted.
Figure 6.3 Experimental time histories of Journal 1 vertical response during start up for a) 0%, b) 20% and c) 40% wear depth. Horizontal response in d) for 0%, e) 20% and f) 40% wear depth.
In Figure 6.3 and Figure 6.4 it is shown that both time histories, simulated and measured, are of the similar progress, having almost equal position inside the radial clearance for the variable wear depths. The journal orbits validate the almost equal “equilibrium position” around which the whirling is developed in each speed. The journal orbits for the time moments around rotational speeds of $\Omega = 50 \, \text{rad/s}$, $\Omega = 100 \, \text{rad/s}$, $\Omega = 150 \, \text{rad/s}$, $\Omega = 200 \, \text{rad/s}$, $\Omega = 250 \, \text{rad/s}$ and $\Omega = 300 \, \text{rad/s}$ are extracted from the startup time histories, and are presented in Figure 6.5 together with the geometry of the worn bearing for each wear depth.
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Figure 6.5 Journal 1 trajectories of experimental rotor bearing system and simulation for various rotational speeds during start up and for variable wear depths. a) 50 rad/s, b) 100 rad/s, c) 150 rad/s, d) 200 rad/s, e) 250 rad/s, f) 300 rad/s.

It is noticed that up to near the critical speed the response amplitude is smoothly increased yielding linear characteristics in the response as also noticed in rotor orbits in Figure 6.5. Since the critical speed is approximated further and passed, the system response becomes with nonlinear characteristics both in experiment and simulation. This fact can be a result not only from bearing properties but also from internal damping tendency to destabilize above critical speed as many authors have proposed. However, as shown in journal orbits obtain nonlinear characteristics in speeds above critical speed.

The fact that the mathematical model can be adapted to the behaviour of the real one in various speeds is the result of nonlinear fluid film consideration and the current combination of continuous rotor with journal bearings. The current simulation converges with the measured
rotor response amplitude in speeds even near critical speed, where different theories yield results even out of radial clearance.

The obvious dive of the journal towards the wear zone is observed as the wear depth is increased. Also, the orbits near resonance ($\omega = 200\text{rad/s}$) are computable, as shown in Figure 6.5d, and this provides the ability of the algorithm to evaluate the rotor response in such cases. If bearing coefficients were used, there would be a difficulty in calculating them under the response of such a great percentage of radial clearance. The current assumption makes the introduction of any fluid film property (stiffness and damping) in the fluid film force feasible and independent of how far the journal whirls from the supposed equilibrium position.

Having a detailed look of Figure 6.5, the measured journal response inside the bearing is in relative good agreement with its computational counterpart in most rotational speeds. In more details, both responses have similar amplitudes within the rotational speed range before and after critical speed. It is worthwhile mentioning that the corresponding responses’ comparisons (see Figure 6.5) may be characterized as preliminary (raw) results. They are mainly presented for pointing out the potentials of the proposed model formulation. More sophisticated convergence between the measured and simulated responses may be obtained via implementing a dedicated model updating procedure, employing the current measurements, the proposed model formulation and a proper nonlinear optimization scheme. Yet, this is beyond the scope of the current study and is left for future research.

Concerning journal orbits of Figure 6.5 the operation eccentricity and the Sommerfeld number can be defined so as to have the information about the bearing load capacity ability of intact as long as worn bearings. The worn-scalloped bearings reduce load capacity and this may yield to contact and scuffing at low speeds.

As it is shown in Figure 6.6 the eccentricity and attitude angle of journal 1 is in a relative good value comparing simulation and measurement considering the fact that the extraction of the journal equilibrium position that corresponds in every speed is very difficult (see Figure 6.5). However an averaging in journal trajectories is a way to yield hypothetic equilibrium positions. The eccentricity ratio of the intact journal bearing obtains values with a maximum of 0.55 with the Sommerfeld number to be at 0.05. The bearing load capacity ability can be reduced more, considering worn bearings, without yielding contact since the eccentricity ratio, of early worn bearings, does not obtain values higher than 0.7 as it is verified from Figure 6.6. As it is shown in Figure 6.6, there is an agreement between simulation and measurement about eccentricity and attitude angle definition, but considering also Figure 6.5 the fact of eccentricity and attitude angle definition is quite difficult since an equilibrium position is not easily defined in
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Each speed. Thus, non monotonic progress of eccentricity and attitude angle through Sommerfeld number variation is noticed (See Figure 6.6).

![Figure 6.6](image)

(a) Journal 1 a) eccentricity ratio and b) attitude angle as a function of Sommerfeld number for variable wear depths.

6.2.1 Time response of worn system with different (larger) slenderness ratio L/D (simulation only)

As in chapter 5 where the effects of crack were investigated additionally in a system of higher slenderness ratio L/D, in this chapter the effects of wear are investigated in the same system, defined in Table 6.2. Since the different slenderness ratio affects the whirling progress it is almost expected that the current system can be useful in order to make some further notifications about the wear effects.

<table>
<thead>
<tr>
<th>Shaft Radius</th>
<th>R = 0.025 m</th>
<th>Material Loss Factor</th>
<th>$\eta = 0.001$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Step Length</td>
<td>L₁ = 1 m</td>
<td>Bearing Length</td>
<td>Lᵇ = 0.025 m</td>
</tr>
<tr>
<td>2nd Step Length</td>
<td>L₂ = 1 m</td>
<td>Bearing Radial Clearance</td>
<td>Cr = 100 μm</td>
</tr>
<tr>
<td>Disk Radius</td>
<td>Rd = 0.19 m</td>
<td>External Load</td>
<td>Eᶠ = (9.81 md) Nᵗ</td>
</tr>
<tr>
<td>Disk Density</td>
<td>$\rho = 7832 \text{ kg/m}^3$</td>
<td>Rotor Static Bowing</td>
<td>$\theta_b = 0.00025$ m</td>
</tr>
<tr>
<td>Disk Width</td>
<td>Lᵈ = 0.022 m</td>
<td>Unbalance Force</td>
<td>$F_u = (e_b m_d \Omega^2)$ Nᵗ</td>
</tr>
<tr>
<td>Oil Viscosity</td>
<td>$\mu = 0.006 \text{ Pa s}$</td>
<td>Disk Mass</td>
<td>$m_d = 19.54$ kg</td>
</tr>
<tr>
<td>Young Modulus</td>
<td>$E = 2.068 \text{ GPa}$</td>
<td>Shaft Density</td>
<td>$\rho = 7832 \text{ kg/m}^3$</td>
</tr>
</tbody>
</table>

Table 6.2 Geometric and physical properties of the rotor bearing system with high L/D ratio
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The system startup is performed from the initial rotational speed $\Omega = 30 \text{ rad} / \text{s}$ to the maximum $\Omega = 100 \text{ rad} / \text{s}$ with an acceleration of $\dot{\Omega} = 4 \text{ rad} / \text{s}^2$, while the sampling frequency is $1 / \Delta t = 800 \text{Samples} / \text{s}$. Note that the sampling frequency that is a significant parameter is a result of various tests in order to make the algorithm computable. A time step of $\Delta t = 0.00125 \text{s}$ is used in all evaluations. The material loss factor is set arbitrary in this low value just to “cut” the infinite response so as to make the startup computable.

In Figure 6.7 it is clear that the wear defect affects the transient time response during start up by introducing an additional resonance during the passage of critical speed. This additional weak resonance is a matter of bearing property shift due to wear that in other words can be said as the further spread of stiffness and damping bearing properties of vertical and horizontal direction. This further anisotropy is translated in this additional resonance that can be expressed more intensively with different geometric properties (for example, the radius of}

\begin{figure}[h]
\centering
\subfloat[][]
\end{figure}
gyration has an important role). In following sections the time histories will be analyzed further in order to investigate the properties in frequency domain.
6.3 Inspection of periodicity and quasi-periodicity

As mentioned above the non-linearity of fluid film forces, in combination with the existence of the wear, assign in rotors response harmonics that each of them can be amplified depending on the defect and the rotational speed.

Figure 6.8 Disk centre experimental time history poincaré maps for wear depth 0% in a) 100 rad/s, b) 200 rad/s, c) 300 rad/s.

Figure 6.9 Disk centre experimental time history poincaré maps for wear depth 20% in a) 100 rad/s, b) 200 rad/s, c) 300 rad/s.

Figure 6.10 Disk centre experimental time history poincaré maps for wear depth 40% in a) 100 rad/s, b) 200 rad/s, c) 300 rad/s.
In accordance of the analysis in Section 5.4 poincaré maps are plotted for the similar rotational speeds and variable wear depths of 20% and 40% as shown in Figures 6.8 to 6.10.

### 6.4 Results for frequency response

The response amplitude is extracted from Figure 6.3 and Figure 6.4 and plotted as a function of rotational speed in Figure 6.11. There is a quite good convergence between experimental and simulative frequency response in the value of critical speed as long as in magnitude of response as it is shown in Figure 6.11 for each case of wear depth. In detail, the intact system critical speed is calculated from Figure 6.11a (vertical plane) and is about 226rad/s in the simulation and 215rad/s in the experiment, thus a convergence of 95%. In the case of 20% wear depth the corresponding critical speed estimation yields the values of 227rad/s and 218rad/s, thus a convergence of 96%. In the case of 40% wear depth the corresponding critical speed estimation yields the values of 217rad/s and 220rad/s, thus a convergence of 98%.

In the horizontal plane, the values of critical speed as long as the convergence become different. Also the experimental horizontal frequency response presents some main differences in relationship with the simulated one providing instabilities in speeds around critical. The intact system critical speed for the horizontal plane is calculated from Figure 6.11b and is about 188rad/s in the simulation and 180rad/s in the experiment, thus a convergence of 95%. In the case of 20% wear depth the corresponding critical speed estimation yields the values of 202rad/s and 179rad/s, thus a convergence of 87%. In the case of 40% wear depth the corresponding critical speed estimation yields the values of
190rad/s and 176rad/s, thus a convergence of 92%. See also Table 6.3 for critical speed values. Note that all critical speeds values are estimated from the graphs in Figure 6.11 and thus the values are not so precise since the frequency response amplitude is not always appropriate for resonance speed definition.

<table>
<thead>
<tr>
<th>Relative Wear Depth</th>
<th>Vertical Plane (Figure 9a) Simulation / Measurement</th>
<th>Horizontal Plane (Figure 9b) Simulation / Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>226 / 215 (95%)</td>
<td>188 / 180 (95%)</td>
</tr>
<tr>
<td>20%</td>
<td>227 / 218 (96%)</td>
<td>202 / 179 (87%)</td>
</tr>
<tr>
<td>40%</td>
<td>217 / 220 (98%)</td>
<td>190 / 176 (92%)</td>
</tr>
</tbody>
</table>

*Table 6.3 Estimated critical speeds of vertical and horizontal plane for various wear depths (Simulation / Measurement (convergence))*

It is quite difficult to define the horizontal plane resonance speed in the cases that wear is introduced and this has to do with the fact that the wear introduces a variation of bearing properties, mainly in horizontal plane, without to be well modelled. Also it is seen that the critical speed cannot become a parameter in which the decrease or increase will be a function of wear depth. It is sure that the wear defect changes the bearing stiffness and damping properties but it cannot be said that decreases or increases their magnitudes, thus the wear defect can be characterized a defect with various and miss understandable symptoms.

The current convergence of computational and experimental critical speed validates that the current model can adapt the system properties in an efficient way that means good definition of bearing stiffness and damping properties as a function of system's response. Different theories assuming linear bearings or even nonlinear bearings including the usage of nonlinear stiffness and damping coefficients cannot leave out the usage of equilibrium position, instead of current theory that assumes no equilibrium position as it happens in many cases of rotational speeds. For example, how could the measured orbit of Figure 6.2.3e be the result of a bearing simulation using equilibrium position, since no steady response is demonstrated in measurement?

Considering the frequency response of Figure 6.11a (measurement, wear depth 0%) there can be a definition of amplification factor that prescribes the current experimental system. As it was declared before, the current systems damping is dual, internal viscous (bearings), and
internal hysteretic (shaft material). So it is very difficult to extract the real loss factor using the
frequency response diagram. However, there can be a definition of Amplification factor using
Figure 6.11a as it is shown in Figure 6.12. The amplification factor is defined and calculated
as $A_f = \frac{Nc1}{(N2 - N1)} = 7.43$ (see Figure 6.12), having a value that corresponds
sufficiently in the API standard 612. According to API standards 612 (Special Purpose Steam
Turbines For Petroleum, Chemical and Gas Industry Services) for lateral analysis, a
separation margin of $SM\% = \left(126 - 6 / (A_f - 3)\right) - 100\%$ of critical speed that
means $24\% Nc1 \approx 52 \text{ rad/s}$ is needed to separate the critical speed from the minimum
operational speed. There are many additional criteria that API standards prompt to concern,
such as maximum vibration amplitude, maximum unbalance and much more, but since the
current system is experimental, there is no need to analyze further the vibration parameters.

![Figure 6.12 Journal 1 horizontal amplitude process in order to define Amplification Factor.](image)
6.4.1 Frequency response of worn system with different (larger) slenderness ratio L/D (simulation only)

The amplitudes of both locations of journal #1 and disc center of system defined in Table 6.2 are expressed as a function of the instantaneous frequency while starting up, as in Figure 6.13, and a slight shift in the critical speed is observed when the wear depth reaches 40%. The critical speed shift is for a rate of 0.5 rad/s, and a first conclusion from this set up is that the effect of wear on the critical speeds can be negligible.

![Figure 6.13 a) Journal 1 and b) Mid-span vertical frequency response through the first critical speed for variable wear depths.](image)

6.5 Results for time – frequency decomposition using CWT

In continue, time – frequency analysis is performed via the Short Time Fourier Transform in the time histories of Journal 1 (simulated and experimental) for variable wear depths in Figure 6.3 and Figure 6.4. The time histories, simulated and measured, are of a time length of 210 sec as mentioned in previous. There is a basic difference in the way the simulated time histories are obtained in respect with the experimental time histories, and this difference is the sampling rate. The sampling rate in simulation is 200 samples/s while the experimental sampling rate is 5000 samples/s. The simulation sampling rate is defined so as to make the simulation code evaluation feasible, since the incorporation of finite difference expressions of various magnitudes demands a time interval that yields stability. This time interval has to be near 0.005s that yields the sampling rate of 200, which is multiple times smaller than the experimental. The fact of low simulating sampling rate unfortunately yields aliasing in time-
frequency decompositions as it will be shown in continue. The experimental sampling rate on the other hand, is quite large yielding large amount of samples. The experimental time histories are down-sampled for this reason so as to reduce the experimental sampling rate in 200 samples/s in order to make both time histories comparative in frequency characteristics.

In current time-frequency decomposition Continuous Wavelet Transform is used producing the scalograms shown in continue. The transformation is preferred from short time Fourier transform and Wigner-Ville since wavelets provide better time localization, and this characteristic is demanded in current problem since additional harmonics are developed in specific time moments and for a specific time extension. It is useful to present the scalogram parameters. The Morlet wavelet is used, the number of analyzed frequencies-scales is 200 and the Morlet wavelet length is 50 samples.

The horizontal response time histories are analyzed using CWT yielding the scalograms of Figure 6.14 for the simulation time histories of Figure 6.4, and of Figure 6.15 for the experimental time histories of Figure 6.3.

![Figure 6.14 Continuous Wavelet Transform (scalogram) for simulated time histories (Horizontal plane) of Journal 1 during start up. a) wear depth 0%, b) wear depth 20%, c) wear depth 40%.](attachment:image1)

![Figure 6.15 Continuous Wavelet Transform (scalogram) for experimental time histories (Horizontal plane) of Journal 1 during start up. a) wear depth 0%, b) wear depth 20%, c) wear depth 40%.](attachment:image2)
In Figure 6.14a the time-frequency decomposition of the intact system yields the main synchronous component as long as the 2XRev and other higher harmonics. In the time moments that the system approaches and passes the critical speed (from about 90s and later) sub-harmonics are developed in various scales-frequencies up to synchronous and higher. These sub-harmonics obtain their higher amplitude near the scales-frequencies of 1/2XRev and 1/3XRev and follow a decrement as the scale-frequency becomes higher from synchronous. However, some of them start at near 80s others in 100s or 120 sec or in general after the approaching of critical speed (case that the nonlinear progression starts to appear). Note that contra-developed harmonics, those from the left to the right, are due to aliasing and are not remarkable but however follow the same progression.

In Figure 6.14b, the case of 20% wear depth, proves the additional development and amplification of sub-harmonics born in the time domain near critical speed in scales-frequencies near 1/2 up to 1/5 of synchronous. However the symptoms noticed in Figure 6.14c for the case of 40% wear depth become a little different and converge to those of the intact system (see Figure 6.17a). This fact can be explained since the high depth of 40% wear offers a different (larger) radial clearance area (see also Figure 7) in which the whirling does not become so effected from the scallops as in case of 20%. The main remark-result of these three scalograms of Figure 6.14 is that sub-harmonics are developed or amplified (if there are already developed) as the wear depth increases during time domain that system approaches and passes the critical speed with the 1/2XRev sub-harmonic to be more sensitive.

The CWT of experimental time histories yields similar results as shown in Figure 6.15 where the symptoms seem to be clearer since the aliasing phenomenon is not presented in such high level as in previous (simulation). In Figure 6.15a there is the clear synchronous response and also sub-harmonics during resonance. As the wear is introduced (see Fig 6.15b) sub-harmonics of near 1/2XRev scale-frequency are developed as the system approaches resonance.

The vertical response time histories were also analyzed using CWT but they did not yield clear differences with wear introduction. This is a matter that proves at least that the bearing properties change in vertical direction is much less severe than the properties in horizontal direction when the wear is introduced. This observation perhaps has to do only with current model geometry only and probably different system geometrical properties could yield differences in vertical direction also. However there is a tendency of a system to provide similar progress also in vertical plane as it is softly presented in Figure 6.16 and Figure 6.17 with Figure 6.17b to have a sensitive difference comparing with Figure 6.17a.
The mechanism that provides the symptoms commented in Figure 6.14 and Figure 6.15 is really a matter and an early conclusion is that the introduction of wear yields in an introduction of additional nonlinearity in fluid film forces, but what could make the specific component of 1/2XRev so sensitive? Considering that the geometry of the worn bearing presents two angles and that the symptoms are presented when the whirling amplitude becomes greater, an answer would be that during the large whirling the fluid film forces have a disturbance two times in each period of spinning as the journal passes through the bearing scallop. Perhaps a future further investigation with a worn bearing with one scallop (only in one side) would arise useful conclusions about the sub-harmonic development.
6.5.1 Time Frequency analysis of worn system response with different (larger) slenderness ratio L/D (simulation only)

Time-frequency analysis is performed via the Short Time Fourier Transform in the time histories of Journal 1 for variable wear depths in Figure 6.16. As shown in the current time-frequency decompositions, the amplitude of the already-appearing (without wear) additional higher harmonics does not present a sensitive increment as the wear depth increases. There is a clear appearance of additional lower and higher harmonics with a task of 1/2XRev at a case of wear depth of 20% when the system is at the 1st critical speed. As the wear depth increases, the 1/2XRev harmonic obtains higher amplitude at the time of critical speed, and an additional harmonic at 3/2XRev and 5/2XRev appear at a wear depth of 40%, always at the time around the critical speed operation. The 1/2XRev harmonic amplitude obtains large amplitude in the case of 40% wear depth, while the other harmonic amplitude continues to have a low magnitude. In all decompositions, there is an obvious bad localization, but this fact does not affect the observations.

The development of 1/2X harmonic is shown better in Figure 6.17 where this specific component is isolated for the above mentioned cases of wear depth including also wear depth 20%. As it is shown in Figure 6.16 the development of 1/2X component due to wear is very sensitive considering the addition of a 20dB Gaussian noise in analyzed signals. The rest developed components of 3/2X and 5/2X are not of similar magnitude and are judged insignificant.

The fact that those harmonic components are introduced at the critical speeds has to do with the large whirling amplitude that the journals obtain at the critical speed. The large whirling amplitude helps the effect of film geometric discontinuity due to wear to become more intense in the journal dynamic oscillation. In other words, if the whirling amplitude is a great percentage of the radial clearance, then the aforementioned harmonics are introduced.

This specific symptom of sub harmonic development due to wear is used further in order to consist a method about wear detection as it will be shown in detail in chapter 7.
Figure 6.18 Log. modulus of the Short Time Fourier Transform (STFT) of the time history through the first critical speed in Journal 1 for a relative wear depth of a) 0%, b) 20%, c) 30% and d) 40%
Figure 6.19 Modulus of STFT for the 1/2X component in Journal 1 response for wear depths of a) 0%, b) 20%, c) 40%. d) Relative Amplitude of 1/2X, 1X, 3/2X and 2X component in Journal 1 response during resonance for wear depths of 20% and 40% during system start up.
6.6 Results for coherency estimation

Considering the unbalance force as an input in the system, since it is the unique excitation force, and the journal response as one of the system outputs, the coherency estimation between them is performed in order to inspect the coherency progress as the wear depth is introduced in the system. Three different steady rotational speeds are investigated: 100rad/s, 200rad/s and 300rad/s and the coherency estimation is plotted in Figure 6.20.
Figure 6.20 Coherency estimation of journal 1 response and disk unbalance force for the rotational speeds of a) 100 rad/s, b) 200 rad/s, c) 300 rad/s.

As it is shown in Figure 20 the shift in coherency of synchronous component is nominal while the coherency of additional components of higher frequency is affected. As it was mentioned through the current chapter the defect of wear does not introduce bearing properties shift in a presumed way. The bearing properties (stiffness and damping) can be affected in various ways as the wear is introduced and this fact yields unforeseen progression of hyper harmonics. In comparison with the coherency shift due to crack introduction (see Figure 5.24) it is clear that the crack introduces a higher nonlinearity in the system than the wear and as it can be seen from Figures 5.24 and 6.20 the coherency decrement in synchronous component is intense in the case of crack in contrast of the progression of synchronous component in worn system where the coherency is not shifted.
6.7 Conclusions for worn model

The effects of worn finite bearings on the dynamics of a continuous rotor were investigated. The following main results are obtained.

1) The wear depth affects the journal equilibrium positions with a higher eccentricity and attitude angle shift as the wear depth increases.

2) The effect of wear at critical speed shifts is complex enough. There is a change in critical speed of a percentage of almost 4% of critical speed but there is not a direct increment or decrement at all cases. The direction of critical speed shift has to do with bearing stiffness properties that were judged unforeseen.

3) In general the wear defect provide the development of lower harmonics (under synchronous). If the slenderness ratio L/D is high (>20) then the development of components near 1/2XRev are amplified. In the case of low L/D ratio (<10) the system obtain amplified sub harmonics of various frequencies.
Chapter 7

The Use of External Excitation in the Crack and Wear Detection

Numerical & Experimental Application

In this chapter the nonlinear rotor bearing system with crack as presented in chapter 1 is used in order to develop the method of detecting the crack presence. In current chapter, computational and experimental time series of the rotor's oscillation, in cases where the system operates in normal conditions are used in order to develop a procedure that identifies the presence of a crack. The finite fluid film bearings as long as the breathing crack can introduce strong nonlinearities and additional coupling that in this work is the tool of crack identification.

The breathing crack affects the coupled response in vertical and horizontal plane when it is open and has no effect while it is closed. Using a horizontal external excitation with specific duration, a method arises this variation of coupling while rotation and the crack presence can be identified. Continuous Wavelet Transform of systems response is used in this chapter so as to well localize the effect of breathing crack coupling in time domain and produce the detection parameter. Note that the method presented is sufficient in identification of almost 10 percent of rotors diameter while can be applied in real time concept.

Additionally, time histories for worn system start up and steady state operations under transient external excitation are analyzed simultaneously in time and frequency domain in order to invest the properties that the wear introduces in the system behavior. Specific components are judged very sensitive to wear development and are used in order to identify the wear extension through the bearing surface.
7.1 Method for Crack Detection

The crack breathing mechanism combined with the ability to well localize coupling existence in time domain is a concept that A. K. Darpe proposed very recently in [215] as a way to detect transverse surface crack in a rotating shafts. In his work, Wavelet transform was used in revealing the transient features of the resonant bending vibrations, which are set up for a short duration of time upon transient torsional excitation. Variation of peak absolute value of wavelet coefficient (of the transient lateral vibration response) with angle at which torsional excitation was applied, was investigated. This last consideration, combined with external excitation force during operation as Ishida and Inoue presented in [216-218], was the basis of the current work’s method in which coupled bending vibration mechanism due to breathing, developed by Chasalevris and Papadopoulos in [219, 220], is used in order to identify an early developed crack. Since the major problem of distinguishing the cracked rotor response from the other nonlinear systems response still remains unsolved, a method is developed so as to achieve the exclusion of the coupling due to crack from the other coupling mechanisms of the system. For this reason, a recent simulation for a rotor bearing system, developed by the authors, is used combining continuous rotor and finite fluid film bearings in a way that nonlinearities are introduced due to bearing forces. By subtracting time histories after crack development from time histories before crack, as Imam et al. in [221], the breathing mechanism is extracted and the additional coupling is investigated according to the external excitation force application. Wavelet transform provides the current method since a well time localization of coupling coefficient is needed. The results encourage the idea of crack identification using exclusively the coupling terms of crack compliance, and not the direct ones, since direct compliance can provoke response similar to other defects or other mechanisms generally.
7.1.1 Application – Results for crack detection

The system response is focused in steady state operation avoiding transient phenomena that does not help in the development of current method. The physical and geometrical properties of the used rotor bearing system (in simulation and in experiment) are given in Table 7.1 (same with Table 6.1). See also Figure 6.1 of current rotor bearing system. The system operates at a steady rotational speed of $\Omega = 500 \text{RPM}$ without this magnitude of rotational speed to be a constraint for the efficiency of the method as it will be shown through the chapter. As long as the system reaches the steady state condition the external sinusoidal force is applied in the location of disk in the horizontal direction (see also Figures 4.8, 4.9 and 4.10 in chapter 4) for a specific amount of time so as to reach a new steady state condition. After an amount of time the external excitation is removed and the system returns to previous steady state condition.

<table>
<thead>
<tr>
<th>Shaft Radius</th>
<th>$R = 0.025$</th>
<th>Material Loss Factor</th>
<th>$\eta = 0.001$ (set)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Step Length</td>
<td>$L_1 = 0.240 m$</td>
<td>Bearing Length</td>
<td>$L_b = 0.025 m$</td>
</tr>
<tr>
<td>2nd Step Length</td>
<td>$L_2 = 0.414 m$</td>
<td>Bearing Radial Clearance</td>
<td>$c_r = 100 \mu m$</td>
</tr>
<tr>
<td>Disk Radius</td>
<td>$R_d = 0.19 m$</td>
<td>External Load</td>
<td>$E_d = W_d \times t$</td>
</tr>
<tr>
<td>Rotor Static bowing</td>
<td>$e_s \approx 0.0015 m$</td>
<td>Unbalance mass</td>
<td>$m_u \approx 0.017 kg$</td>
</tr>
<tr>
<td>Shaft/Disk Density</td>
<td>$\rho = 7832 \text{kg/m}^3$</td>
<td>Oil Viscosity</td>
<td>$\mu \approx 0.013 \text{PaS}$</td>
</tr>
<tr>
<td>Disk Width</td>
<td>$L_d = 0.022 m$</td>
<td>Young Modulus</td>
<td>$E = 2.068 \text{GPa}$</td>
</tr>
</tbody>
</table>

Table 7.1 Geometric and physical properties of the current experimental rotor bearing system

Only the vertical response is used in this chapter since the properties and the effect in vertical response have to be used for the current method. The steady state response in vertical direction is acquired and plotted in Figure 7.1 for three periods of rotation ($3T \approx 1800$ samples since sample rate is $S_r = 5000 \text{samples/s}$) together with the corresponding analytical response of simulation. Note that since the displacement sensors yield the response in Volts and since the relative amplitude is demanded for the progress of the method, the experimental time histories are of magnitude presented in Volts. When the horizontal external excitation is introduced in the system with the frequency of $\Omega_{EX} = 4000 \text{ RPM}$ the vertical response becomes as in Figure 7.2. It is clear that the excitation frequency is introduced in the signal especially in the case of simulation through the mechanism of coupling of the system due to bearings and crack. Note that the real excitation force is not absolutely horizontal due to
manufacturing defects, it is not always applied in the center of the shaft as in simulation and also it is not an absolutely sinusoidal cause of the fact that the whirling amplitude (elastic due to unbalance and static due to bowing) of the rotor changes the gap between coil and shaft yielding a variation of the electromagnetic force out of absolute sinus. The aforementioned reasons can provoke an additional structural (geometric) coupling but the consideration of the method can leave these parameters out without affecting its robustness as it will be shown in continue. Also note that the magnitude of electromagnetic force cannot be measured but is estimated around the 10% of total systems weight as Ishida in [218] prompts.

![Figure 7.1](image1.png)  ![Figure 7.2](image2.png)

Figure 7.1 Vertical response of disc center in rotor speed $\Omega = 500\text{RPM}$ for various cases of crack depth without external excitation. (a) Measurement, (b) simulation.

![Figure 8.1](image3.png)  ![Figure 8.2](image4.png)

Figure 7.2 Vertical response of disc center in rotor speed $\Omega = 500\text{RPM}$ for various cases of crack depth during external excitation of $\Omega_{EX} = 4000\text{ RPM}$. (a) Measurement, (b) simulation.
In continue, and in order to extract the cracked system response, the signals of response with external excitation (see Figure 7.2) are subtracted from each other as the crack depth increases. To explain further the signal of Figure 7.2a of cut depth 0% is subtracted from the signal of cut depth 20%. The resulting signal is this of Figure 7.3a. Note that the case of cut depth 0% in Figure 7.3 represents a measurement made in a time moment of operation between the initial measurement of the intact system and of the measurement of the 20% cracked system. This measurement represents a situation that the system remains intact but there is a change is physical properties of the system such as lubricant viscosity shift due to extended lubricant usage, rotor bowing shift due to thermal stresses during operation and many others.

![Figure 7.3 Difference (cracked-intact) of signals acquired in Figure 7.2. (a) Measurement, (b) simulation.](image)

The signal of Figure 7.3 contains the coupling phenomenon due to crack since the other parameters of coupling are left out using the subtraction. It can be seen also that the amplitude of difference signals varies with time since the coupling phenomenon is not of same intense during rotation and this because of the fact that the coupling obtains its maximum the specific time moment that the crack obtains the maximum asymmetry in regard to excitation plane. In Figure 7.3b it is clear that the coupling is present in time moments that the crack is totally open (samples around 400, 1000, 1600). In order to isolate the component of frequency 4000RPM (external excitation frequency) a continuous wavelet transform (CWT) is made in the difference signal of the various cases of crack depth. The coefficients of wavelet decomposition are presented in the scalograms of Figures 7.4 and 7.5. It is useful to present the scalogram parameters. The “Morlet” wavelet is used, the number of analyzed frequencies-scales is 5160 and the time interval is $dt = 0.0002$. Also the relationship between
scaling and pseudo frequency is \( F_s = F_c / (dt F_r) \) with \( F_s \) to be the scale, \( F_c = 0.8125 \text{Hz} \) to be the Morlet wavelet central frequency and \( F_p \) to be the pseudo frequency. In the scalograms of Figures 7.4 and 7.5 the scales of interest are the scale 488 (pseudo frequency 500RPM) and the scale 61 (pseudo frequency 4000RPM).

![Figure 7.4 CWT (Continuous Wavelet Transform) of difference in Figure 7.3 for the case of crack depth 20%. a) experiment and b) simulation. 2D views in (c) and (d) correspondingly.](image)

a) experiment and b) simulation. 2D views in (c) and (d) correspondingly.
Figure 7.5 CWT (Continuous Wavelet Transform) of difference in Figure 7.3 for the case of crack depth 40%.

(a) experiment and (b) simulation. 2D views in (c) and (d) correspondingly.

From scalograms in Figures 7.4 and 7.5 the coefficient of scale 61 is extracted and plotted in Figure 7.6. Note that in Figure 7.6a the 200 initial samples together with the 200 last samples are not plotted because they yield large amplitude due to CWT algorithm and are not proper to be plotted together with the rest 1400.

The wavelet coefficient of scale corresponding to external excitation frequency is proper to present the coupling due to crack during the rotation of the shaft. It contains only one frequency (4000RPM) and its amplitude is well localized in the time moments that the coupling exists or not. The coupling presence during rotation is a function of crack rotational angle and in Figure 7.6b it is clearly shown that the coupling becomes intense in the moments that the crack is totally open as in times around samples 400, 1000, 1600 and this fact makes the detection of crack feasible since only the defect of crack can yield this dynamic coupling.
In the experimental case the variation of amplitude of wavelet coefficient is also noticed during rotation but in not such a clear way as in simulation and this due to the fact that the experimental crack (cut) remains open during rotation and does not breathes as the crack does in simulation. Thus the coupling due to cut exists in the major part of time needed for an entire rotation. However, the current wavelet coefficient is judged very sensitive in crack depth variation and can be used for crack detection even of cracks of 20% depth as shown in Figure 7.6.

![Graphs showing wavelet coefficient variation](image)

**Figure 7.6** Extraction of wavelet coefficient of Scale 61 (Pseudo frequency 4000RPM). a) Experiment and b) simulation.

The robustness of the method is under considerations for crack depths above 20%. For deeper cracks this specific wavelet localization at the time when the crack opens, betrays the crack presence. As shown in Figure 7.6 the coupling is observed for crack depth even 20%.

The proposed method can be developed in the following steps making the procedure more definite:

1. Set the rotor bearing system in steady state operation with a whirling / spinning speed $\Omega$.
2. With the certainty that there is no crack a number of periods (an amount of 500) are acquired in the vertical plane without the external excitation in horizontal plane.
3. An equal number of periods (an amount of 500) are acquired in the vertical plane with the external excitation in horizontal plane to be present.
4. After a long time of operation we repeat steps 2 and 3.
5. We subtract the signals acquired in step 4 from the corresponding signals acquired in steps 2 and 3.

6. A Continuous Wavelet Transform is made on the two signals. The two signals are the difference without excitation and the difference with excitation.

7. Extract the wavelet coefficients of the scale that corresponds to the frequency of excitation for both signals.

8. Focus in coefficients of step 7 in time domain before and after the excitation intro.

9. If the two signals have differences then a crack of at least 20% depth is present.

The entire procedure for crack detection is presented as a flowchart in Figure 7.7 so as to clearly define the steps needed.

The magnitude of external excitation frequency is a matter of consideration. The external excitation frequency is chosen considering some parameters that have to be mentioned. First of all the excitation frequency has to be several time larger (at least 8) than synchronous rotational speed of the rotor and this because the crack has to receive at least 8 pulses so as to be excited in the 8 basic time moments that has rotational angle of 0°, 45°, 90°, 135°, 180°, 225°, 270° and 315°. So it is clear that the more the pulses are the more possible is to “catch” the crack in open form. On the other hand the additional response due to external excitation force follows the rule of (high frequency)-(low magnitude) or (low frequency)-(large magnitude) and this fact has to be considered because the larger the amplitude is the higher signal to noise ratio (SNR) is achieved. Considering the above parameters it can be said that the “perfect” excitation frequency has to be high and near resonance so as to provoke large additional amplitude. The large additional amplitude provoked from external excitation must not be confused or compared with the respective synchronous amplitude level of the critical speed response of the system. The philosophy of the method is NOT TO DISTURB the system and this can be achieved if the resonance due to external excitation is of 2nd or even 3rd eigenfrequency regarding the location the electromagnet exists so as not to be located in eigenmode nodal point. With this concept in mind the system can be excited with the double excitation frequency of 8000RPM so as to produce an external excitation with frequency near to second eigenfrequency that for the current system is about 7500RPM. In Figure 7.8 the final wavelet coefficient with scale 30 (8000RPM) is plotted for variable crack depths showing that the crack depth of even 20% can be also detected with a higher excitation frequency.

Figure 7.7 Flow chart of the detection method.

Figure 7.8 a) Measured vertical response during external excitation with $\Omega_{EX} = 8000$ RPM. b) Wavelet coefficient of scale 30 (8000RPM). (Experimental)

Different operational conditions for the rotor bearing system are assumed in Figure 7.9 where the wavelet coefficient of scale 21 (11500RPM) is plotted after the application of the method.
for rotational speed of the rotor in $\Omega=1500\text{RPM}$ (3 times higher than in previous) and external excitation frequency in $\Omega_{\text{EX}} = 11500 \text{ RPM}$. Again, the crack of depth 20% can be detected.

**Figure 7.9** Wavelet coefficient of scale 21 (11500RPM) for the case of operational conditions with rotational speed $\Omega=1500\text{RPM}$ and external excitation frequency $\Omega_{\text{EX}} = 11500 \text{ RPM}$. (Experimental)

**Application of the method in bearing measurements**

The applicability of the method in journal response measurements makes the current method useful in real machines where measurements in locations between bearings cannot be made since the machine monitoring can be made only with journal measurements. Also, if the method was efficient with measurements near crack it wouldn’t be useful since the crack location in real situation cannot be predicted. For this reason the method is applied in journal response measurements (experimentally and analytically) so as to give a sense of the method applicability.

The operational conditions of rotational speed in $\Omega = 500\text{RPM}$ and external excitation frequency $\Omega_{\text{EX}} = 4000 \text{ RPM}$ are used to measure and to calculate the initial signals. It is interesting to show what happens inside the bearing when external excitation is applied in a different location in the rotor span. In Figure 7.10 the journal orbits with and without excitation are presented. In continue and in Figure 7.11 the measured and simulated vertical response
of journal #1 is plotted with and without external excitation for the cases of crack depths 0%, 20% and 40%. As it is shown in Figure 7.11 the effect of crack in journal response is not as visible as in the case of disc center measurements (see Figures 7.1 and 7.2). However, the applicability of the method is feasible since the difference signals in Figure 7.12 prove that the effect of the crack can be measured and isolated even in journal response.

![Figure 7.10 Measured Journal #1 orbits without and with external excitation \( \Omega_{EX} = 4000 \) RPM during steady rotational speed operation of \( \Omega = 500 \) RPM. a) experiment b) simulation.](image1)

![Figure 7.11 Journal #1 vertical response for variable crack depths and external excitation of \( \Omega_{EX} = 4000 \) RPM during steady rotational speed operation of \( \Omega = 500 \) RPM. a) experiment b) simulation.](image2)
In Figure 7.13 the wavelet coefficient of scale 61 (4000RPM) is plotted and it can be seen that there is a sensitivity that provides crack detection from journal measurements. Note that the simulation results provide also the crack effect along the entire system and this fact is a result of current assumption of rotor bearing system simulation (chapter 1). The information about the local flexibility discontinuity due to crack can be obtained from any point through the entire length of the system and this property of the simulation (as long as of the physical system) can lead to proposals about crack location detection especially if the system is symmetric regarding to bearings. The concept of crack location detection seems up to now to be a little bit discouraging since the magnitude of amplification of signal coefficients due to coupling has to do with a lot of parameters such as rotor mass near the position of measurement and near the position that the crack appears. Also the operational conditions have a great influent in the magnitude of crack-provoked symptoms in the response. Additionally the physical properties of the system, if for some reason become altered, can affect the coefficient used for detection. All these aforementioned reasons together with many others make the crack location a matter of future research.

![Figure 7.12 Difference (cracked-intact) of signals acquired in Figure 7.11. (a) Measurement, (b) simulation.](image-url)
7.1.2 Conclusions for crack detection method

From the current treatment with the coupling phenomenon in this rotor bearing system, some conclusions are just repeated since they have been made from various researchers and some of them introduce a different point of view in the so called coupling behavior. More specifically, the conclusions that arise from the current procedure can be summed as follows:

I) The additional response due to crack presence contains two main components: The first is the component due to direct compliance and the second is the component due to coupling compliance. The first component includes most of the entire 2X but also higher harmonics due to stiffness alteration while breathing but the second contains most of all, the response of the other main plane where an excitation exists. This means that the component due to coupling can transfer the excitation frequency in the plane where no excitation exists. This phenomenon happens only in a specific moment: when the crack is open. It is not a requirement to be fully open, but when it is fully opened it is better. Here something must be clear: The totally open crack must provoke by its form the total asymmetry of the cracked section in respect to external excitation direction. To make this clearer, if the external excitation was in vertical direction no coupling would appear since a total open crack makes the cracked section absolutely symmetric to vertical plane. In this case of course there would be a coupling due to other coupling compliance terms but in a 90 degree rotation (semi closed).
II) The fact that the additional response due to crack is extracted as the difference of the response with the crack and without crack is not enough to conclude that really there is a crack. There are lots of mechanisms that develop an additional response with the most percentage of 2X frequency. The extracted signal must be examined carefully in time domain with respect to “what happens in the other plane” and “what is the form of the crack” in this time simultaneously. The fact that there is not the ability to know which is the situation of the crack, since we don’t know if there is a crack, is overtaken by continuous excitation of much higher from the synchronous frequency. This means that the excitation must make a max and a minimum effort many times while the crack is totally opened. So the fact of perfect localization of signal components in time domain, using wavelets, gives the ability to “catch” the specific time that the excitation frequency intrudes rapidly in the “difference” signal, with this rapid introduction to happen one single time during driving period.

III) Using the above considerations a crack can be identified since no other mechanisms can provoke similar observations. Additionally, under the ability of exciting the system in different locations in between bearings the coupling phenomenon will be observed in the locations nearer to the crack. Also the phenomenon is acquired better from the sensors near to the crack that means that if a crack is near to a bearing the journal response must be acquired so as to identify the crack. This mechanism can be used in order to have a sensation of the crack location. In what has with crack depth to do, it is obvious that crack depths near 20% of rotors radius are not severe and there is no need to have a sense for the crack depth since the depth is in this extension. However it will be a problem if a deeper crack is presented rapidly. This will result in fault diagnosing since a crack of depth higher than 20% of rotor radius will be concerned as a crack of maximum depth 20% and this will lead in faulty results. Since a rapid crack development is not common the method leads to a good estimation of initial crack depths.

7.2 Method for Wear Detection

According to chapter 6, time histories of the response during the system startup were analyzed in order to reveal additional harmonics developed due to wear. These specific components are judged very sensitive to wear development and are used in this chapter in order to identify the wear presence through the bearing surface. However there is not always the ability to have time histories through resonance, since the passage through critical speed has to be rapid and not as this of the current rotational acceleration. For this reason the idea of exciting the system with an additional external excitation while operating in steady state is
introduced in this work so as to inspect the additional harmonic development with the amplitude of resonance to be of much lower value. In the disc location of the rotor a horizontal sinusoidal excitation of linear frequency (chirp) and steady amplitude of almost 10% of systems weight is applied. The external excitation force properties are shown in Figure 7.14. These harmonics can be developed during the steady state operation, under horizontal electromagnetic external excitation (as in section of crack detection) with this way of operation to be more proper for diagnosis since, as shown in the section of crack detection, does not affects the system main operation that has to be of constant rotational speed.

The notifications are made in two systems, one of low and one of large slenderness ratio L/D, as presented also in chapters 5 and 6. A further notification is made in system of large slenderness ratio about the development of 1/2X harmonic since its sensitivity to wear development makes this fact useful for wear detection.

![Figure 7.14 External Excitation Chirp in time domain and Wigner-Ville decomposition](image)
7.2.1 Application – Results for wear detection

The system of Figure 6.1 with the properties of Table 7.1 is set in the operational condition of rotational speed $\Omega = 1500$ RPM and transient external excitation (chirp) in order to achieve the sub harmonic development due to wear under the usage of external excitation force. After the encouraging results about sub harmonic development in worn system start up in chapter 6, the use of external excitation as a concept of wear detection comes up since the external excitation can develop the nonlinear resonances of a defected system.

![Figure 7.15 Scalograms of journal 1 vertical response time histories for $\Omega = 1500$RPM and a) wear depth 0%, b) wear depth 20% and c) wear depth 40% (experimental)](image)

![Figure 7.16 Scalograms of journal 1 horizontal response time histories for $\Omega = 1500$RPM and a) wear depth 0%, b) wear depth 20% and c) wear depth 40% (experimental)](image)

A slight development of sub harmonic components of various scales can be noticed in scalograms of Figure 7.15 for vertical response and in Figure 7.16 for horizontal response at time moments near systems’ first natural frequency (40 secs).

Time Frequency analysis of worn system response with different (larger) slenderness ratio L/D (simulation only)

The effects of wear under external excitation are investigated additionally in a system, defined in Table 6.2. Since the different slenderness ratio affects the whirling progress it is almost expected that the current system can be useful in order to make some further notifications about the wear effects.

In spectrograms of Figure 7.19 it is shown that as the wear is introduced in the system the component of half frequency of external excitation is developed during resonance. The external excitation is transient (chirp) and its components are isolated and plotted in 3D plots in Figure 7.19 while the amplitudes of components of 1/2X, 1X, 3/2X and 2X are plotted as a function of wear depth in Figure 7.20. It can be said that the component of frequency 3/2X is as sensitive as component 1/2X and this is a difference in symptoms of worn systems of large and small ratio L/D.
Figure 7.19 a) Magnitude of STFT for journal 1 horizontal response under external excitation for wear depth 0%. 1/2X component magnitude in (b). c) Magnitude of STFT for Journal 1 horizontal response under external excitation for wear depth 20%. 1/2X component magnitude in (d). e) Magnitude of STFT for Journal 1 horizontal response under external excitation for wear depth 40%. 1/2X component magnitude in (f).
7.2.2 Conclusions for wear detection

The main aim of this section of wear detection was to investigate the characteristics of the response of the system under external excitation in time and frequency domain simultaneously. Since the defect of wear was introduced the results were focused in the development of additional components and especially of this with frequency half of synchronous.

The synchronous whirling frequency is the frequency of the component with the higher amplitude. In the case that no external excitation is present, the synchronous whirling frequency is equal to the rotational frequency. In the case that external excitation is present; the synchronous whirling frequency is equal to the external excitation frequency. A sub-synchronous component is sensitively amplified as the wear depth begins to exist near the time of resonance. However there are system properties that help this development in a narrow time spreading before and after resonance and other system properties accumulate this development at resonance moment with this observation to be clear. The wear depth that is judged able to provoke these symptoms so as detection to be provided is about 20% of radial clearance.

Future work based on these observations would possibly develop the method of quantization of wear by normalizing the additional components with respect to intact system response.
General Conclusions and Future Trends

The current dissertation includes general and specific results in various domains such as the nonlinear simulation of rotor bearing systems, the effects of crack and wear in the systems response, and the ability of detecting the current defects with the presented methods. At the end of each chapter the most important notifications and conclusions were presented in a detailed form, yielding the contribution and the usage of each progress in current work.

At this point, with a glance through the entire process, concluding remarks can be made expressing the main results of current dissertation. The concluding remarks about the defect symptoms are divided in these that have with rotor crack and bearing wear to do. In rotor crack symptoms, the current dissertation provides the coupled bending vibrations due to breathing crack as a reliable tool in order to detect the crack presence using external excitation. The nominative demands for reliable detection are an excitation force of almost 10% of system's total mass and a crack of depth 5% of rotor diameter.

The breathing crack form, as it is simulated in current dissertation, is a function of systems response and provides higher nonlinearity than the simulation of periodically varying stiffness coefficients due to crack and this fact has as a reason the lower coherency between responses in bearing and mid-span especially when the crack depth exceeds the 40% of rotor radius.

The effects of worn finite bearings on the dynamics of a continuous rotor were investigated and the following main results are obtained. The wear depth affects the journal equilibrium positions with a higher eccentricity and attitude angle shift as the wear depth increases. The effect of wear at critical speed shifts is complex enough. There is a change in critical speed of a percentage of almost 4% of critical speed but there is not a direct increment or decrement at all cases. The direction of critical speed shift has to do with bearing stiffness properties that were judged unforeseen. In general the wear defect provide the development of lower harmonics (under synchronous). If the slenderness ratio L/D is high (>20) then the development of components near 1/2XRev are amplified. In the case of low L/D ratio (<10) the system obtain amplified sub harmonics of various frequencies.

The current dissertation can be characterized as a continue in trends of the early and late past in further (more complete) modeling of rotor bearing systems with the usual and often presented defects of crack and wear, including the nonlinearities that produce special types of oscillations, yielding symptoms able to provide defect detection as long as better convergence of simulation to real system in significant parameters (magnitudes) during operation.
It is a fact that the simulation concept presented in current dissertation offers a relative good convergence to the real system especially in the significant magnitudes of critical speed and additional harmonic development during resonance. The additional harmonic development was a result yielded by the current nonlinearity assumption (nonlinearity introduced by the crack and the bearings) providing at last the concept of fault detection, but additionally the concept of revealing defect symptoms using external excitation devices is a matter that nowadays is more and more under consideration. Further, the defect simulation provided very well the defect symptoms especially in the case of crack simulation where the coupling compliance simulation became the most significant reason about the success of crack detection. The simulation code developed during this dissertation is able to produce nonlinear oscillations provided due to defect presence but also from the intact systems operation during resonance and sometimes after critical speed (higher frequencies) due to instabilities caused from internal (hysteretic) damping. The ability of current code to produce instabilities at operational speeds higher than critical speed coincides with the opinion that internal (hysteretic) damping acts in the system by destabilizing it only after critical speed and not at all speeds as many authors have proposed in the past.

With the aforementioned concepts under consideration together with the very latest trends in rotor dynamics literature a lot of matters are under consideration and could obtain great interest in the future in order not only to efficiently design rotor bearing systems but also to faithfully monitor large rotating machinery under the view of economy and safety.

The treatment of internal (hysteretic) damping, as an additional nonlinearity in the system, using variable loss factor as a function of developed stresses in the entire rotor could be a simulation that probably would provide defect symptoms directly related with the developed response during resonance. In other words, a better simulation of systems internal (hysteretic) damping could yield the exact response amplitude at the critical speed and under the consideration that at this operational condition the stresses are high, even a small open crack could probably yield a shift in response amplitude. However, the current matter seem to be a quite complex mathematic problem since the nonlinear partial differential equations (i.e. Rayleigh) have to be treated using hyper geometric series and additionally the nonlinearity introduced by the variable loss factor has to be treated as a feed back of systems response and systems stresses.

The use of external excitation force as a manner to yield symptoms of defects can be further investigated especially in the concept of crack detection. The up to now well modeled crack behavior, not only achieved in current simulation but also in the huge amount of papers about crack breathing behavior published from many researchers, can obtain a different role. In
brief, the function of local crack compliance during rotation can be used as an excitation so as to yield special response characteristics only when a crack exists. An extended idea in this matter would be to program some special excitation functions, even using wavelet functions, so as to produce the crack symptoms in the response in a much more clear and efficient way. With the proper excitation the cracked rotor probably will react in such a manner that crack detection will be more efficient.

It is a fact that even with the best rotor bearing system design, the existence of control devices makes the operation of rotating machines more efficient. At the moment, the use of magnetic bearings seems to be the mostly used devices that offer rotor mounting and system controlling at the same time. The concept of magnetic bearings is a current trend in rotor dynamics for various reasons and the combination of continuous rotor with magnetic bearings can be treated as an ability to investigate crack symptoms under bearing excitation since the magnetic bearings are able to obtain this role.

The simulation of the wear defect borrowed from the literature for the demands of current work can surely be improved in the future. The geometric symmetry of wear zone assumed in the current wear model can perhaps judged as a little bit unrealistic since the journal oscillations appear always in an attitude angle with respect to bearing axis of symmetry. A future simulation under the consideration of fluid film mechanics in heavy loaded bearings incorporating perhaps contact mechanics also, would probably yield a different wear model, more realistic, providing perhaps symptoms closer to reality.

There are of course many future concepts already provided from many researchers around the world in various orientations such as nanotechnology and rotor dynamics, atomistic simulation in fracture mechanics, composite rotors, smart fluids (lubricants) and others but it is understandable that the current dissertation stands in the field of traditional rotor dynamic treatment, thus the suggestions of future work made in previous paragraphs does not include the concepts mentioned before.
References


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Appendix A

Short Information about Wavelet Transformation

The wavelet function that is used to transform a signal \( x(t) \) at scale \( s \) and location \( b \) is defined as:

\[
\psi_{s,b}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t-b}{s}\right)
\]  

(A.1)

A natural way to sample the parameters \( s \) and \( b \) is to use a logarithmic discretization of \( s \) scale and link this to the size of steps taken between \( b \) locations. To link to \( s \) we move in discrete steps to each location \( b \) which is proportional to \( s \) scale. The function of the wavelet has the form:

\[
\psi_{m,n}(t) = \frac{1}{\sqrt{s_0^n}} \psi\left(\frac{t-n \cdot b_0 \cdot s_0^n}{s_0^n}\right), \quad s_0 > 1, \ b_0 > 0, \ m, n \in \mathbb{Z}^*
\]  

(A.2)

The wavelet transform of \( x(t) \) using wavelets of the form above is:

\[
T_{m,n} = \int_{-\infty}^{\infty} x(t) \cdot \frac{1}{s_0^{n/2}} \psi\left(s_0^{-n} \cdot t - n \cdot b_0\right) dt
\]  

(A.3)

Common choices for discrete wavelet parameters \( s_0 \) and \( b_0 \) are 2 and 1 respectively. This power of two logarithmic scaling of both the dilation and translation steps is known as the dyadic grid arrangement. Discrete wavelets are associated with scaling functions and their dilation equations. The scaling equation which has the form \( \phi_{m,n}(t) = 2^{-m/2} \cdot \phi(2^{-m} \cdot t - n) \) describes the scaling function \( \phi(t) \) in terms of contracted and shifted versions of itself with equation \( \phi(t) = \sum c_k \cdot \phi(2^{-k} t - k) \) (A.3).

Scaling coefficients \( c_k \) must satisfy the constraint \( \sum c_k = 2 \) (A.3) (integrate two sides of (A.2)) and in order to create an orthogonal system Equation (A.3) is required.

\[
\sum c_k \cdot c_{k+2^l} = \begin{cases} 2 & \text{if } k' = 0 \\ 0 & \text{otherwise} \end{cases}
\]  

(A.3)

Also the smoothness of the wavelet is associated with a moment condition which can be expressed as
\[
\sum_{k=0}^{N-1} (-1)^k \cdot c_k \cdot k^n = 0 .
\] (A.4)

In this procedure wavelets that have a finite number of scaling coefficients \( N \) are considered.

For this case the wavelet function is defined as

\[
\psi(t) = \sum_{k} (-1)^k c_{N-1-k} \cdot \phi(2 \cdot t - k) .
\] (A.5)

Here a Daubechie wavelet \( DN \) with three scaling coefficients (D3) is used. This wavelet has a support length equal to 2, \( (N - 1) \). From Equation (A.2) the scaling function for a three coefficient wavelet is

\[
\phi(t) = c_0 \cdot \phi(2t) + c_1 \cdot \phi(2t - 1) + c_2 \cdot \phi(2t - 2)
\] (A.6)

and from Equation (A.5) the wavelet function is

\[
\psi(t) = -c_2 \cdot \phi(2t - 1) + c_1 \cdot \phi(2t - 2) - c_0 \cdot \phi(2t - 3) .
\] (A.7)

To find the values of scaling coefficients for the D3 wavelet, Equations (A.2), (A.3) and (A.4) are used. The solution of the system of equations below gives the values of scaling coefficients.

\[
\begin{align*}
\begin{cases}
c_0 + c_1 + c_2 = 2 \\
c_0^2 + c_1^2 + c_2^2 = 2 \\
c_0 - c_1 + c_2 = 0
\end{cases}
\Rightarrow \begin{cases}
c_0 = \frac{5 - 3\sqrt{2}}{7}, c_1 = \frac{6 + 2\sqrt{2}}{7}, c_2 = \frac{3 + \sqrt{2}}{7} \\
c_0 = \frac{5 + 3\sqrt{2}}{7}, c_1 = \frac{6 - 2\sqrt{2}}{7}, c_2 = \frac{3 - \sqrt{2}}{7}
\end{cases}
\] (A.8)

The first solution of (A.8) leads to \( \phi(t) \) and the second to \( \phi(-t) \). The first set is adopted and the scaling function for the D3 wavelet is computed using Equation (A.6) with a change shown in Equation (A.9).

\[
\phi_j(t) = c_0 \cdot \phi_{j-1}(2t) + c_1 \cdot \phi_{j-1}(2t - 1) + c_2 \cdot \phi_{j-1}(2t - 2)
\] (A.9)

The subscript number \( j \) is the iteration number. Choosing an initial shape for \( \phi(t) \) Equation (A.9) is iterated until \( \phi_j(t) = \phi_{j-1}(t) \) and the wavelet is defined directly using Equation (A.7).
Appendix B

Simulation of Coupled Bending Vibrations of a Beam

The sub matrices of Matrix $P$ are defined as follows:

\[
\begin{bmatrix}
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & k_1 & 0 & k_2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\cosh(Lk_1) & \sinh(Lk_1) & \cos(Lk_1) & \sin(Lk_1) & -\cosh(Lk_1) & -\sinh(Lk_1) & -\cos(Lk_1) & -\sin(Lk_1) \\
-k_1\cosh(Lk_1) & -k_1\sinh(Lk_1) & -k_1\cos(Lk_1) & -k_1\sin(Lk_1) & k_1\cosh(Lk_1) & k_1\sinh(Lk_1) & k_1\cos(Lk_1) & k_1\sin(Lk_1) \\
-k_2\cosh(Lk_2) & -k_2\sinh(Lk_2) & -k_2\cos(Lk_2) & -k_2\sin(Lk_2) & k_2\cosh(Lk_2) & k_2\sinh(Lk_2) & k_2\cos(Lk_2) & k_2\sin(Lk_2) \\
\ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
-k_8\cosh(Lk_8) & -k_8\sinh(Lk_8) & -k_8\cos(Lk_8) & -k_8\sin(Lk_8) & k_8\cosh(Lk_8) & k_8\sinh(Lk_8) & k_8\cos(Lk_8) & k_8\sin(Lk_8) \\
\end{bmatrix}
\]

\[
O_{8,1} = E_k y^2 \cosh(Lk_1)c_{551} + k_1 \sinh(Lk_1),
\]

\[
O_{8,2} = E_k y^2 \sin(Lk_1)c_{552} + k_1 \cosh(Lk_1),
\]

\[
O_{8,3} = -E_k y^2 \cos(Lk_1)c_{551} - k_1 \sin(Lk_1),
\]

\[
O_{8,4} = -E_k y^2 \sin(Lk_1)c_{552} + k_1 \cos(Lk_1),
\]

\[
O_{8,5} = -k_1 \sinh(Lk_1),
\]

\[
O_{8,6} = k_1 \cosh(Lk_1),
\]

\[
O_{8,7} = k_1 \sin(Lk_1),
\]

\[
O_{8,8} = -k_1 \cosh(Lk_1),
\]

\[
O_{12,5} = E_k y^2 \cosh(Lk_2)c_{551} + k_1 \sinh(Lk_2),
\]

\[
O_{12,6} = E_k y^2 \sinh(Lk_2)c_{552} + k_1 \cosh(Lk_2),
\]

\[
O_{12,7} = -E_k y^2 \cos(Lk_2)c_{551} - k_1 \sinh(Lk_2),
\]

\[
O_{12,8} = -E_k y^2 \sinh(Lk_2)c_{552} + k_1 \cos(Lk_2),
\]

\[
O_{8,13} = E_k y^2 \cosh(Lk_1)c_{541},
\]

\[
O_{8,14} = E_k y^2 \sinh(Lk_1)c_{542},
\]

\[
O_{8,15} = -E_k y^2 \cos(Lk_1)c_{543},
\]

\[
O_{8,16} = -E_k y^2 \sin(Lk_1)c_{544},
\]

\[
O_{12,17} = E_k y^2 \cosh(Lk_2)c_{541},
\]

\[
O_{12,18} = E_k y^2 \sinh(Lk_2)c_{542},
\]

\[
O_{12,19} = -E_k y^2 \cos(Lk_2)c_{543},
\]

\[
O_{12,20} = -E_k y^2 \sin(Lk_2)c_{544},
\]

\[
-285-
\]
\[ O_{12,19} = -Elk_y^2 \cos(L_2 k_z) c_{44_2}, \quad O_{12,20} = -Elk_y^2 \sin(L_2 k_z) c_{44_2} \]

\[ N = \begin{bmatrix} \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix} \]

\[ O_{20,1} = Elk_y^2 \cosh(L_1 k_y) c_{45_5}, \quad O_{20,2} = Elk_y^2 \sinh(L_1 k_y) c_{45_5}, \quad O_{20,3} = -Elk_y^2 \cos(L_1 k_y) c_{45_5}, \]

\[ O_{20,4} = -Elk_y^2 \sin(L_1 k_y) c_{45_5}, \quad O_{24,5} = Elk_y^2 \cosh(L_2 k_y) c_{45_2}, \quad O_{24,6} = Elk_y^2 \sinh(L_2 k_y) c_{45_2}, \]

\[ O_{24,7} = -Elk_y^2 \cos(L_2 k_y) c_{45_2}, \quad O_{24,8} = -Elk_y^2 \sin(L_2 k_y) c_{45_2} \]

\[ D = \begin{bmatrix} \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix} \]

\[ O_{20,13} = Elk_y^2 \cosh(L_1 k_y) c_{44_4} + k_z \sin(L_1 k_z), \quad O_{20,14} = Elk_y^2 \sinh(L_1 k_y) c_{44_4} + k_z \cosh(L_1 k_z) \]

\[ O_{20,15} = -Elk_y^2 \cos(L_1 k_y) c_{44_4} - k_z \sin(L_1 k_z), \quad O_{20,16} = -Elk_y^2 \sin(L_1 k_y) c_{44_4} + k_z \cos(L_1 k_z) \]

\[ O_{20,17} = -k_z \sin(L_1 k_z), \quad O_{20,18} = -k_z \cosh(L_1 k_z), \quad O_{20,19} = k_z \sin(L_1 k_z), \quad O_{20,20} = -k_z \cos(L_1 k_z) \]

\[ O_{24,17} = Elk_y^2 \cosh(L_2 k_y) c_{44_2} + k_z \sin(L_2 k_z), \quad O_{24,18} = Elk_y^2 \sinh(L_2 k_y) c_{44_2} + k_z \cosh(L_2 k_z) \]

\[ O_{24,19} = -Elk_y^2 \cos(L_2 k_y) c_{44_2} - k_z \sin(L_2 k_z), \quad O_{24,20} = -Elk_y^2 \sin(L_2 k_y) c_{44_2} + k_z \cos(L_2 k_z) \]

\[ O_{24,21} = -k_z \sin(L_2 k_z), \quad O_{24,22} = -k_z \cosh(L_2 k_z), \quad O_{24,23} = k_z \sin(L_2 k_z), \quad O_{24,24} = -k_z \cos(L_2 k_z) \]

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Appendix C

Notes
Rotating machine catastrophic failure due to excessive vibrations. (Courtesy of Bently Nevada Corporation.)
CARL GUSTAF PATRIC DE LAVAL (1845–1913)

De Laval was born in Sweden. He studied in the Uppsala Technical University, where he was awarded a Ph.D. in 1872 in mathematics and natural sciences. His first contributions were in metallurgy: the extraction of copper and the iron-nickel-phosphorus alloys. He further contributed substantial developments in metallurgy, such as the arc melting and lead and zinc metallurgy. Then he developed the centrifugal separator for the production of butter. He developed a working steam turbine and experimented at high speeds, up to 30,000 rpm. This won him the title “Man of High Speeds.” More important, he observed and found engineering solutions for most problems of rotor dynamics, such as whirl, critical speeds, and accelerating beyond the critical speed.
AUGUST FÖPPL (1854–1924)

Born in Gross-Umstadt in the family of a doctor, Föppl studied structural engineering in Darmstadt, Stuttgart, and Karlsruhe, mostly under Mohr. For many years he taught at trade schools and did research on structures and later on Maxwell's theory of electricity. He succeeded Bauschinger in the chair of mechanics at Munich. His textbook Lectures in Mechanics had a lasting impact in the field. He did experimental work on fatigue and stress concentration. He was the first to give a satisfactory theory for shaft whirling and did extensive experimental work on the dynamics of the single-disk rotor, which he called the "de Laval rotor."
STEPHEN P. TIMOSHENKO (1878–1972)

Born and educated in Russia, where he held professorships in Petrograd and Kiev. Timoshenko left Russia and came to the United States after he taught for a short time at Belgrade and studied for short periods with Prandtl. He had a profound influence in the United States, where, after a short tenure with Westinghouse, he became a professor at Michigan and at Stanford. Having contributed to the theory of elasticity and vibration, he was instrumental in the development of mechanics in the United States. He published several books in mechanics.
The symbol $1X$, $2X$ or in general $(Number)X$ expresses frequency of value $(Number)X$ (rotational speed). i.e. if a rotor is spinning with a speed 500RPM then the frequency $2X$ is 1000RPM. Usually the expression of frequencies can be also as $1X$Rev or $2X$Rev where “Rev” stands for “frequency of revolution”. Note that there can be any real number in the expression i.e. $1.47X$Rev.

JULES-HENRI POINCARÉ (1854–1912)

Poincaré was born at Nancy to a prominent family. He graduated from the Ecole Polytechnique with a degree in mining engineering and was employed by the Department of Mines for the remainder of his life. He earned a doctorate from the University of Paris, where he held several professorships. He wrote very extensively, more than any mathematician of our century, in a variety of areas, including differential equations, probability, and celestial mechanics. He is considered the founder of topology.

He did not stay long in one area: “He was a conqueror, not a colonist.” He died at the age of 58.
The fact that fluid film forces are a function of rotors response adds in the system an additional nonlinearity to this of crack.

The assumption of short or long bearing yields analytical solutions of Reynolds equation. If $L$ is the bearing length through axial axis and $D$ is the bearing diameter, a bearing is characterized as “Long” if $L/D > 1$ and as “Short” if $L/D < 0.5$. Each type yield different properties in what has to do with stability, impedance due to misaligning, loading capacity, viscous damping, friction coefficient, heat transfer and other significant properties in proper operation. Thus the bearings consist a machine element strongly related with the reliability and the efficiency of the entire system.

Current work incorporates the hysteretic internal damping (material damping) with constant loss factor and internal viscous damping due to fluid film bearings. No contribution is made about the property of hysteretic damping and its effects in systems response. Hysteretic damping is incorporated only in order to give the ability of critical speed response evaluation, that is a matter of great importance for the appropriate operation (amplification factor) as long as for the extraction of useful results for crack and wear effects.

The internal hysteretic damping is modeled with the use of complex Young modulus $E' = E + i\eta$ but the equations of motion are later divided in Real and Imaginary part so as to obtain the ability of evaluating fluid film forces.
The modified Newton Raphson method is used here because of the expected difficulty in defining proper initial values. As the system “starts” the modified method is needed but in steady state becomes in its traditional form providing low evaluation time cost.

The crack depth $a$ (see Nomenclature) is a parameter always referred in relationship with the shaft radius. It is assumed to be small, that means $a/R < 0.5$. This assumption follows the consideration that small cracks are formed more reliably as closed in region of suppression stresses and open in region of tension stresses than the deep cracks do. Also the consideration of small crack depths provides the concept of “early diagnosis”.

Here, with the term “discrete” the variable but non continuous rotational angle change of the transverse crack around the axial axis of the shaft is described. The rotational angle of the crack does not varies during vibration but remains constant until the response and other vibration characteristics are extracted for the current value of rotational angle. When this is done, then, a new rotational angle is assumed and the same process is repeated up to the point that the rotational angle covers an entire rotation of 360 degrees.
Large Turbine Rotor. Photo from MAN TURBO AG.

The shaft L/D ratio (L: Shaft length from one bearing to another, D: Shaft diameter) met in real turbomachines is usually under 20 demanding the transverse shear effect incorporation (4,22)
26 Gyroscopic effects appear due to continuous mass shaft rotation as long as due to rotation of the disc mounted on any point along the axial axis of the shaft.

27 Axial torque due to power transmission is regarded only as a variable considered in the solution towards generality. Axial torque in the entire work is set zero “0” since torsional vibrations is not the concept. However, the axial torque has important effects to vibratory behavior (4).

28 The term “coupled” refers, here, to the coupled Rayleigh equations of motion and comprises one of the three main reasons of coupling in current work modeling: Crack, Bearings, and Shaft.

19 As it will be presented in detail in Chapter 3, the crack is introduced in the system with compliance-depended boundary conditions in the slope property. During rotation, the crack breathes and the compliances change thus there is a slope boundary conditions alteration.

20 Full stiffness bearings are “virtually” the joints in both ends of the rotor. This consideration enables the bearing stiffness and damping effect neglection.

21 Poincaré maps are used in current work in order to make observations of quasi periodic motions. The periodicity or aperiodicity of time histories was observed in various cases.

22 Time – Frequency analysis is performed in this work using Short Time Fourier decomposition and Continuous Wavelet Transform according to the case. Wigner-Ville decomposition is also used in few cases.
Bearing wear is introduced in the finite bearings using the Dufrane model. The wear defect is incorporated just as a geometric property and no discussion is made in this work about the wear development mechanism or other relative to wear matters (material properties, surfaces in contact etc).

Poincaré maps are used in current work in order to make observations of quasi periodic motions. The periodicity or aperiodicity of time histories was observed in various cases.

Time – Frequency analysis is performed in this work using Short Time Fourier decomposition and Continuous Wavelet Transform according to the case. Wigner-Ville decomposition is also used in few cases.

http://scholar.google.gr/scholar?num=100&hl=el&lr=&cites=8480226228910731737
“Music melodies meet their perfection in the sound of machines, because the machine sounds like an orchestra under the conduction of physical laws, laws that a God defined” – A. Chasalevris