Value-at-Risk in the Presence of Asymmetry: The FIGARGH model

Ilias Salatas

A dissertation submitted in partial fulfillment of the requirements for the degree of Master of Science in Applied Economics & Data Analysis

School of Business Administration
Department of Economics

Master of Science in
“Applied Economics and Data Analysis”

Patras August 2017
Three-member Dissertation Committee

Research Supervisor: Athanasios Tsaganos Assistant Professor

Dissertation Committee Member: Ioannis Venetis Associate Professor

Dissertation Committee Member: Dimitrios Tzelepis Assistant Professor

The present dissertation entitled

«Value-at-Risk in the Presence of Asymmetry: The FIGARGH model»

was submitted by Ilias Salatas, Sid 1057154, in partial fulfillment of the requirements for the degree of Master of Science in «Applied Economics & Data Analysis» at the University of Patras and was approved by the Dissertation Committee Members.
I would like to dedicate my dissertation to all who are fighting for a better Future.
Acknowledgments

I would like to thank my supervisor, Mr Athanasios Tsaganos for his support, his collaboration, his help and his encouraging advice during realizing this thesis. The interaction with Mr Tsaganos was not only productive but also a great great pleasure. Also the members of Three-member Dissertation Committee Mr Ioannis Venetis and Mr Dimitrios Tzelepis for the interest in time series and data analysis they stimulated and inspired me during our collaboration in present MSc. I appreciate their help and teaching more than I could imagine as they provided many valuable research ideas and suggestions for next academic steps. I heavily benefited from all Teachers of Master of Science in Applied Economics & Data Analysis of University of Patras and I wish to them the best and to continue trying for the best. Also Mrs Eleni Karfaki and finally and most of all, I would like to thank my family and my girlfriend Betty. They always supported and encouraged my in every craziness I do and I know they will keep doing so. They had patience and gave me the mental support, optimism, and retreat that enabled me to successfully finish my studies.
Summary

Most widely used financial instrument to monitor risk is VaR (Value at Risk). VaR is a standard tool for measuring potential risk of economic losses in financial markets as a single number. There are several ways to calculate this single number, one among others, according to parametric method, is with the help of ARCH and GARCH models.

The ARCH and GARCH models became important tools in the analysis of time series data, particularly in financial applications. These models are especially useful when the goal of the study is to analyze and forecast volatility. The long memory of FIGARCH (Fractionally Integrated Generalized Autoregressive Conditional Heteroscedastic) model allows it to be a better candidate than other heteroscedastic models for evaluating asymmetry features and long memory in the volatility of Markets such as Athens Stock Market (ATHEX). This thesis will try to examine the concept of long persistence in order to combine the VaR calculation of Athens Stock Exchange General index (ASE) for a given time horizon and given confidence level using FIGARCH (1,d,1) model with Student-t, and skewed Student-t distribution for errors.

Keywords: VaR, FIGARCH models, Volatility, Long Memory, ARFIMA, GARCH models

---

1Exibition of persistance. Thus strong shocks seem to impact future volatility for long periods of time.
Περίληψη

Το πιο διαδεδομένο μέτρο ποσοτικοποίησης του κινδύνου σήμερα είναι η Λέξια σε Κίνδυνο ή VaR (Value at Risk). Η VaR είναι ένα τυπικό εργαλείο που μας βοηθά να προσεγγίσουμε πιθανούς κινδύνους οικονομικών απωλειών από τη δραστηριότητα μας στις οικονομικές αγορές. Υπάρχουν πολλές μέθοδοι υπολογισμού αυτού του αριθμού, ένας ανάμεσα σε αυτούς είναι με τη βοήθεια των μοντέλων ARCH και GARCH. Τα μοντέλα ARCH και GARCH έχουν αναχθεί σε πολύ σημαντικά εργαλεία στην ανάλυση δεδομένων χρονοσειρών, ειδικά οικονομικών χρονοσειρών. Τα υποδείγματα αυτά είναι ιδιαίτερα χρήσιμα όταν ο σκοπός της μελέτης είναι η ανάλυση και η πρόβλεψη της μεταβλητότητας της αγοράς (volatility). Το μοντέλο FIGARCH (Fractionally Integrated Generalized Autoregressive Conditional Heteroscedastic) λόγω της ισχυρής μνήμης φύσης του φαίνεται πως είναι καλύτερος υποψήφιος από άλλα ετεροσκεδαστικά μοντέλα για την εκτίμηση ασυμμετρικών χαρακτηριστικών και ισχυρής μνήμης στη μεταβλητότητα αγοράς όπως το Χρηματιστήριο Αθηνών. Σε αυτή τη μελέτη θα προσπαθήσουμε να εξετάσουμε τον ορισμό της επιστολικής μνήμης για τον υπολογισμό VaR του Γενικού Δείκτη Τιμών του Ελληνικού Χρηματιστηρίου Αθηνών για δεδομένο χρονικό διάστημα και επιπέδου εμπιστοσύνης με τη βοήθεια του FIGARCH (1,d,1) μοντέλου και με κατανομή Student-t, και skewed Student-t των σφαλμάτων.

Λέξεις κλειδιών: Λέξια σε Κίνδυνο, FIGARCH μοντέλα, Μεταβλητότητα, Ισχυρή Μνήμη, ARFIMA, GARCH μοντέλα

2 Επιδεικνύεται επιμονή. Ισχυρά σόκ τείνουν να επηρεάζουν τη μελλοντική μεταβλητότητα για μεγάλο χρονικό διάστημα.
# Contents

1 Introduction 1

2 General Remarks 6
   2.1 Risk ........................................ 6
   2.1.1 Financial Risks .......................... 8
   2.1.2 Risk and Reward ......................... 13
   2.2 Volatility .................................. 14
   2.2.1 Measures ................................ 14
   2.3 VaR ......................................... 16
   2.3.1 VaR Methods ................------------ 19
   2.3.2 Limitations of VaR ..................... 26

3 PARAMETRIC MODELS 27
   3.0.1 ARIMA ........................... 31
   3.0.2 ARCH GARCH ......................... 32
   3.0.3 LONG MEMORY IN VOLATILITY ARFIMA and FIGARCH 37

4 EMPIRICAL FINDINGS 42
   4.1 Preliminary Analysis of Data ............ 42
   4.2 Long Memory and Asymmetry for RASEIN 46

5 Conclusions 49

Bibliography-References 51

List of Figures
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Volatility of Bitcoin 2010-2016</td>
<td>1</td>
</tr>
<tr>
<td>2.1</td>
<td>Systematic And Unsystematic Risk</td>
<td>9</td>
</tr>
<tr>
<td>2.2</td>
<td>Definition Of VaR</td>
<td>18</td>
</tr>
<tr>
<td>3.1</td>
<td>Garch(1,1)</td>
<td>35</td>
</tr>
<tr>
<td>3.2</td>
<td>ACF PACF X</td>
<td>36</td>
</tr>
<tr>
<td>3.3</td>
<td>ACF PACF X²</td>
<td>36</td>
</tr>
<tr>
<td>4.1</td>
<td>ATHENS GENERAL STOCK MARKET INDEX 5/2/88 to 31/7/17</td>
<td>43</td>
</tr>
<tr>
<td>4.2</td>
<td>LOG RETURNS OF ASE INDEX</td>
<td>43</td>
</tr>
<tr>
<td>4.3</td>
<td>HISTOGRAM AND AGAINST NORMALITY TEST</td>
<td>44</td>
</tr>
</tbody>
</table>

**List of Tables**
4.1 summary statistics of RASEIN . . . . . . . . . . . . . . . . . . . . . . 44
4.2 Unit Root Tests . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 45
4.3 Estimation Results of FIGARCH Models for the RASEIN . . . . . 47
Chapter 1

Introduction

Figure 1.1: Volatility of Bitcoin 2010-2016

Note that in graph (1.1) we see the volatility of Bitcoin. Bitcoin is widely considered a highly volatile asset. Both the US Consumer Financial Protection Bureau\(^1\) and US National Association of Attorneys General\(^2\) have cited volatility as a significant risk of holding bitcoin. But what precisely do we mean by volatility

\(^1\)https://www.consumerfinance.gov
\(^2\)http://www.naag.org
in Economics and Finance? After all, as Granger pointed out “volatility is not directly observed or publicly recorded.” The term has been used by a lot of researchers in different contexts and it is not clear that they all refer to the same attribute.

To understand volatility we need to consider how one can measure this thing from observed data by linking it to concepts of uncertainty and risk. Volatility, as being used in everyday language, refers to variability in prices or returns such as stock returns and exchange rate returns. The most popular measure of this type of volatility is the standard deviation. Other commonly used measures include the interquartile range and the mean absolute return.

So the asset and stock market volatility is of paramount importance to many parts like market practitioners, institutions including banks, portfolio managers, regulators, policy makers etc. The practitioner is concerned about stock market volatility because it affects asset pricing and risk, whereas the policy maker attempts to curb excessive volatility to ensure financial and macroeconomic stability. The market crash in October 1987, recent 2008 crisis, financial regional crises in emerging markets as Greece, and trading losses of well-known financial institutions have led regulators and supervisory committees to favor using reliable quantitative techniques to appraise possible losses.

Value at Risk (VaR) has become one of the most popular risk measures for quantifying and controlling the market risk of a portfolio by institutions (see Jorion, 1996, 2000) and is presented as an instrument which reduces the risk and defines its scale. It is really crucial to understand the mechanisms that create behaviors in the financial world, and most importantly, the methods which allow somehow to a possible reduction of their negative effects. Indeed, it is important

---

4 The VaR tries to answer the following question: What is the predicted financial loss over a given time period, with a given level of confidence? The VaR of a portfolio is the maximum loss it may suffer in the course of a certain holding period, which is usually one day or ten days. Hence, the VaR of an investor’s portfolio is the maximum amount of money that can be lost in the short-term.
to have the knowledge of the instruments in which to invest, but even more it is important to know how to deal with risk reduction which appears always with these type financial investments. Any measure, including the Value at Risk, may not only allow for a full exploration of knowledge about the mechanisms that create the financial markets but, more importantly can be used as a tool to fight over the negative consequences of our decisions. Hence, modeling VaR has become an appealing research area. In the last few years, great efforts have been put to develop the best model for VaR computation.\footnote{Beder (1995), Hendricks (1996), and Marshall and Siegel (1997) discuss the importance of the underlying models for estimating VaR.}

VaR can be estimated parametrically (for example, variance-covariance VaR or delta-gamma VaR), nonparametrically (for example, historical simulation VaR) or semiparametrically (Extreme Value Theory, CAViaR and quasi-maximum likelihood GARCH). Nonparametric methods of VaR estimation are discussed in Markovich and Novak\footnote{Markovich, N. (2007), Nonparametric analysis of univariate heavy-tailed data, Wiley Novak, S.Y. (2011). Extreme value methods with applications to finance.}. A comparison of a number of strategies for VaR prediction is given in Kuester et al\footnote{Kuester, Keith; Mittnik, Stefan; Paolella, Marc (2006). "Value-at-Risk Prediction: A Comparison of Alternative Strategies". Journal of Financial Econometrics.}.

Several methods have been developed for measuring parametrically VaR. The RiskMetrics model developed by the risk management group at J.P. Morgan in 1994 has become a benchmark for measuring market risk. Also Conditional volatility models are applied very often in investigations of stock exchange time series. GARCH models are the most important class of these models. The volatility models that have been extensively used in the literature are Autoregressive Conditional heteroscedastic (ARCH) model of Engle (1982), Generalized Autoregressive Conditional Heteroscedastic (GARCH) model of Bollerslev (1986), Integrated GARCH (IGARCH) model of Engle and Bollerslev (1986) and Fractionally Integrated GARCH (FIGARCH) model of Baillie, Bollerslev, and Mikkelsen (1996). Baillie et al. (1996) proposed the Fractionally Integrated Generalized Autoregressive Conditional Heteroscedasticity (FIGARCH) model by generalizing the IGARCH model.
to allow for persistence in the conditional variance. It is worthwhile to investigate whether long memory property of volatility in financial time series can affect the measurement of market risk.

Long memory is the exhibition of persistence. Thus strong shocks seem to impact future volatility for long periods of time; more analytically, the presence of long memory in return and volatility implies that there exist dependencies between distant observations. Hence, the market does not immediately respond to information flowing into the financial markets, but reacts to it gradually over time. Long memory in returns and volatility are also found to have significant effect on the pricing of financial derivatives as well as forecasting market volatility.

The primary purpose of introducing FIGARCH model was to develop a more flexible class of processes for the conditional variance, that are capable of explaining and representing the observed temporal dependencies in financial market volatility. In particular, the FIGARCH model allows only a slow hyperbolic rate of decay for the lagged squared or absolute innovations in the conditional variance function. This model can accommodate the time dependence of the variance and a leptokurtic unconditional distribution for the returns with a long memory behaviour for the conditional variances.

In every case, an efficient quantitative tool for modeling stock market volatility is needed to minimize the risk of inaccurate measurement. In this regard, researchers continue to search for the best volatility model that is able to capture various stylized facts associated with market volatilities.

This Thesis has been framed in such a way to be comprehensible piece by piece. So after introduction in chapter one the second chapter is used to present and define some General meanings for Risk, Volatility and how all are connected to the meaning of VaR. Then we discuss about the parametric VaR calculation methods and especially the parametric methods based in GARCH models but our main concern is the presentation of the FIGARCH model. The remainder of the study is organized as follows: Chapter 4 provides the statistical characteristics of
data set used and reports the empirical results after fitting FIGARCH. Dataset is a timeseries that reflect logarithmic daily returns of Athens Stockmarket General Index from 5/2/88 till 31/7/2017 a total of 7536 observations. Conclusions are presented in Section 5.
Chapter 2

General Remarks

We will present some fundamental meanings to built step by step the knowledge basis so we can understand and then make an empirical application of the FI-GARCH model.

2.1 Risk

Risk is part of human life and history. Comes from Latin word *risco* a combination of the words *re-* (back) and *secare* (cut). Is an expression attributed to sailors and used when their ships were near to rocks and reefs.

From the moment we get up in the morning, drive or take public transportation to get to school or to work until we get back into our beds, we are exposed to risks of different degrees. The study of risk is fascinating because some of this risk facing may be completely voluntary, we seek out some risks on our own (speeding on the highways or gambling, for instance) and enjoy them. While some of these risks may seem trivial, others have big importance in the way we live our lives. On a loftier note, it can be argued that every major advance in human civilization, from the caveman’s tools invention to gene therapy and space conquer, has been made possible because someone was willing to take a risk and challenge the status quo.

Given the ubiquity of risk in almost every human activity, it is surprising how little consensus there is about how to define risk. The first discussion focused on
2.1 Risk

the distinction between risk that could be quantified objectively and subjective risk. In 1921, Frank Knight summarized the difference between risk and uncertainty thus\(^1\): "... Uncertainty must be taken in a sense radically distinct from the familiar notion of Risk, from which it has never been properly separated. ... The essential fact is that 'risk' means in some cases a quantity susceptible of measurement, while at other times it is something distinctly not of this character; and there are far-reaching and crucial differences in the bearings of the phenomena depending on which of the two is really present and operating. ... It will appear that a measurable uncertainty, or "risk" proper, as we shall use the term, is so far different from an un-measurable one that it is not in effect an uncertainty at all." So, Knight defined only quantifiable uncertainty to be risk and provided an example of two individuals drawing from an urn of red and black balls. The first individual is ignorant of the numbers of each color whereas the second individual is aware that there are three red balls for each black ball. As a result the second individual estimates (correctly) the probability of drawing a red ball to be 75% but the first operates under the misperception that there is a 50% chance of drawing a red ball. Knight argues that the second individual is exposed to risk but that the first suffers from ignorance.

The point on whether uncertainty is subjective or objective seems misplaced. It is true that a measurable risk is easier to be insured but we do care about all uncertainty in general, whether measurable or not. In a paper on defining risk, Holton (2004)\(^2\) argues that there are two ingredients that are needed for risk to exist. The first is uncertainty about the potential outcomes from an experiment and the other is that the outcomes have to matter in terms of providing utility. He notes, for instance, that a person jumping out of an airplane without a parachute faces no risk since he is certain to die (no uncertainty) and that drawing balls out of an urn does not expose one to risk since one's well being or wealth is unaffected by whether a red or a black ball is drawn. Of course, attaching different

---

\(^1\)Knight, F.H., 1921, Risk, Uncertainty and Profit, New York Hart, Schaffner and Marx

monetary values to red and black balls would convert this activity to a risky one. Also according to Holton, risk has two components: exposure, and uncertainty. If we swim in shark-infested waters, we are exposed to bodily injury or death from a shark attack. We are uncertain because we don’t know if we will be attacked. Being both exposed and uncertain, we face risk. Risk metrics typically take one of three forms:

- those that quantify exposure.
- those that quantify uncertainty.
- those that quantify exposure and uncertainty in some combined manner.

Probability of rain is a risk metric that only quantifies uncertainty. It does not address our exposure to rain, which depends upon whether or not we have outdoor plans. Credit exposure is a risk metric that only quantifies exposure. It indicates how much money we might lose if a counterparty were to default. It says nothing about our uncertainty as to whether or not the counterparty will default. Risk metrics that quantify uncertainty—either alone or in combination with exposure—are usually probabilistic. Many summarize risk with a parameter of some probability distribution. Standard deviation of tomorrow’s spot price of copper is a risk metric that quantifies uncertainty. It does so with a standard deviation. Average highway deaths per passenger-mile is a risk metric that quantifies uncertainty and exposure. We may interpret it as reflecting the mean of a probability distribution. 

*Risk is the exposure to an uncertain factor with possibility to be caused undesirable results or risk is a consequence of action taken in spite of uncertainty.*

### 2.1.1 Financial Risks

Financial Risk is any type of risk associated with the chance that the return achieved on an investment will be different from that expected. This includes economic transactions of course. Often it is understood to include only downside risk but in modern portfolio theory this is not absolute see short investment positions.
A science has evolved around managing market and financial risk under the general title of modern portfolio theory initiated by Dr. Harry Markowitz in 1952 with his article, "Portfolio Selection". In modern portfolio theory, the variance (or standard deviation) of a portfolio is used as the definition of risk.

**Systematic And Unsystematic Risk**

![Diagram of Systematic And Unsystematic Risk](image)

**Figure 2.1: Systematic And Unsystematic Risk**

In Finance Risk Management Total Financial risk is composed by the Unsystematic risk, also known as "diversifiable risk" "specific risk," or "residual risk," is the type of uncertainty that comes with the company or industry you invest in, better with the stocks of those companies we have invested in. Unsystematic risk can be reduced through diversification. For example, news that is specific to a small number of stocks, such as a sudden strike by the employees of a company you have shares in, is considered to be unsystematic risk. And on the other hand is the Systematic risk, known as "market risk" or "un-diversifiable risk", is the uncertainty inherent to the entire market or entire market segment. Also referred to as volatility, systematic risk consists of the day-to-day fluctuations in a stock’s price.

---

2.1 Risk

Volatility is a measure of risk because it refers to the behavior, or "temperament," of your investment rather than the reason for this behavior. Because market movement is the reason why people can make money from stocks, volatility is essential for returns, and the more unstable the investment the more chance there is that it will experience a dramatic change in either direction.

Interest rates, recession and wars all represent sources of systematic risk because they affect the entire market and cannot be avoided through diversification. Systematic risk can be mitigated only by being hedged.

Systematic risk underlies all other investment risks. If there is inflation, you can invest in securities in inflation-resistant economic sectors. If interest rates are high, you can sell your utility stocks and move into newly issued bonds. However, if the entire economy underperforms, then the best you can do is attempt to find investments that will weather the storm better than the broader market. Popular examples are defensive industry stocks, for example, or bearish options strategies.

Beta is a measure of the volatility, or systematic risk, of a security or a portfolio in comparison to the market as a whole. In other words, beta gives a sense of a stock’s market risk compared to the greater market. Beta is also used to compare a stock’s market risk to that of other stocks. Investment analysts use the Greek letter $\beta$ to represent beta. Beta is used in the capital asset pricing model (CAPM).

Other Types of Risk

Except Market risk and Diversifiable risk we discussed, there are other types of risks not so fundamental but yet mentionable, let’s look at more specific types of risk, particularly when we talk about stocks and bonds.

- Asset-backed risk. Risk that the changes in one or more assets that support an asset-backed security will significantly impact the value of the supported security. Risks include interest rate, term modification, and prepayment risk.
2.1 Risk

- Credit risk. Credit risk, also called default risk, is the risk associated with a borrower going into default (not making payments as promised). Investor losses include lost principal and interest, decreased cash flow, and increased collection costs. An investor can also assume credit risk through direct or indirect use of leverage. For example, an investor may purchase an investment using margin. Or an investment may directly or indirectly use or rely on repo, forward commitment, or derivative instruments.

- Foreign investment risk. Risk of rapid and extreme changes in value due to: smaller markets; differing accounting, reporting, or auditing standards; nationalization, expropriation or confiscatory taxation; economic conflict; or political or diplomatic changes. Valuation, liquidity, and regulatory issues may also add to foreign investment risk.

- Liquidity risk. This is the risk that a given security or asset cannot be traded quickly enough in the market to prevent a loss (or make the required profit). There are two types of liquidity risk:
  a) Asset liquidity - An asset cannot be sold due to lack of liquidity in the market.
  b) Funding liquidity - Risk that liabilities: Cannot be met when they fall due, Can only be met at an uneconomic price, Can be name-specific or systemic.

- Market risk factors. The four standard market risk factors are equity risk, interest rate risk, currency risk, and commodity risk:
  Equity risk is the risk that stock prices in general (not related to a particular company or industry) or the implied volatility will change.
  Interest rate risk is the risk that interest rates or the implied volatility will change.
  Currency risk is the risk that foreign exchange rates or the implied volatility will change, which affects, for example, the value of an asset held in that
Commodity risk is the risk that commodity prices (e.g., corn, copper, crude oil) or implied volatility will change.

- **Operational risk.** Operational risk is "the risk of a change in value caused by the fact that actual losses, incurred for inadequate or failed internal processes, people and systems, or from external events (including legal risk), differ from the expected losses" in other words as long as people, systems and processes remain imperfect, operational risk cannot be fully eliminated. This definition, adopted by the European union Solvency II Directive for insurers, is a variation from that adopted in the Basel II regulations for banks. In October 2014, the Basel Committee on Banking Supervision proposed a revision to its operational risk capital framework that sets out a new standardized approach to replace the basic indicator approach and the standardized approach for calculating operational risk capital.

- **Some Other risks**

  Reputational risk: Reputational risk, often called reputation risk, is a risk of loss resulting from damages to a firm’s reputation.

  Legal risk: According to Basel II classified Legal risk as a subset of Operational Risk in 2003. There is no standard definition, but there are at least two primary/secondary definition sets in circulation.¹ ²

  IT risk: Information technology risk, or IT risk, IT-related risk, is any risk related to information technology.

---

¹Mcormick, R. 2004 Legal risk is the risk of loss to an institution which is primarily caused by: (a) a defective transaction; or (b) a claim (including a defense to a claim or a counterclaim) being made or some other event occurring which results in a liability for the institution or other loss (for example, as a result of the termination of a contract) or; (c) failing to take appropriate measures to protect assets (for example, intellectual property) owned by the institution; or (d) change in law.

²Johnson and Swanson. 2007 The expenses of litigation of a company. Whalley, M. 2016 Legal risk is the risk of financial or reputational loss that can result from lack of awareness or misunderstanding of, ambiguity in, or reckless indifference to, the way law and regulation apply to your business, its relationships, processes, products and services. Tsui TC. 2013 The cost and loss of income caused by legal uncertainty, multiplied by possibility of the individual event or legal environment as a whole. One of the most obvious legal risks of doing business not mentioned in the above definitions is the risk of arrest and prosecution.
Model risk: In finance, model risk is the risk of loss resulting from using insufficiently accurate models to make decisions, originally and frequently in the context of valuing financial securities.

Country Risk: Country risk refers to the risk that a country won’t be able to honor its financial commitments. When a country defaults on its obligations.

Political Risk: Political risk represents the financial risk that a country’s government will suddenly change its policies. This is a major reason why developing countries lack foreign investment.

As we can see, there are several types of risk that a smart investor should consider and pay careful attention to.

### 2.1.2 Risk and Reward

The rephrased Greek old saying “if you don’t wet your feet you will not eat fish” has a logical base. Those who desire large rewards have to be willing to expose themselves to considerable risk. The link between risk and return is most visible when making investment choices! Stocks are riskier than bonds, but generate higher returns over long periods. It is less visible but just as important when making career choices, a job in sales and trading at an investment bank may be more lucrative than a corporate finance job at a corporation but it does come with a greater likelihood that you will be laid off if you don’t produce results.

Not surprisingly, therefore, the decisions on how much risk to take and what type of risks to take are critical to the success of a business. A business that decides to protect itself against all risk is unlikely to generate much upside for its owners, but a business that exposes itself to the wrong types of risk may be even worse off, though, since it is more likely to be damaged than helped by the risk exposure. In short, the essence of good management is making the right choices when it comes to dealing with different risks.
2.2 Volatility

As we discussed before volatility is a measure of risk, not risk itself. Volatility is a statistical measure of the dispersion of returns for a given security or market index. Volatility does not measure the direction of price changes, merely their dispersion. Ways volatility can be measured is by using the standard deviation or variance between returns from that same security or market index, also we can use Betas, R-Squared, Alphas etc. Commonly, the higher the volatility, the riskier the security. In other words, volatility refers to the amount of uncertainty or risk about the size of changes in a security’s value. A higher volatility means that a security’s value can potentially be spread out over a larger range of values. This means that the price of the security can change dramatically over a short time period in either direction, so here we can see how time as a parameter is inserted to our discussion. A lower volatility means that a security’s value does not fluctuate dramatically, but changes in value at a steady pace over a period of time.

2.2.1 Measures

We mention beta $\beta$ as one measure of volatility, actually is a measure of the relative volatility of a particular stock to the market. A beta approximates the overall volatility of a security’s returns against the returns of a relevant benchmark. For example, a stock with a beta value of 1.2 has historically moved 120% for every 100% move in the benchmark, where is based on price level. Conversely, a stock with a beta of .8 has historically moved 80% for every 100% move in the underlying index. Some other volatility measurements are:

- The R-squared of a fund advises investors if the beta of a mutual fund is measured against an appropriate benchmark.

- Alpha which measures how much of the extra risk posed by factors other than market volatility helped the fund outperform its corresponding benchmark.
2.2 Volatility

- Standard deviation.

While beta determines the volatility (or risk) of a fund in comparison to that of its index, standard deviation determines the volatility of a fund according to the disparity of its returns over a period of time.

Standard deviation is simply defined as the square root of the average squared deviation of the data from its mean. In finance, volatility is often used to refer to \( \sigma \) or variance \( \sigma^2 \), computed from a set of observations as

\[
\hat{\sigma}^2 = \frac{1}{N-1} \sum_{t=1}^{N} (R_t - \bar{R})^2
\]  

(2.1)

Where \( \bar{R} \) is the mean return. The sample standard deviation statistic \( \hat{\sigma} \) is a distribution free parameter representing the second moment characteristic of the sample. Only when \( \sigma \) is attached to a standard distribution, such as a normal distribution or a t one can the required probability density and cumulative probability density be derived analytically.

While this statistic is relatively easy to calculate, the assumptions behind its interpretation are more complex, which in turn raises concern about its accuracy. As a result, there is a certain level of skepticism surrounding its validity as an accurate measure of risk as Troy Adkins says \(^6\).

In order for standard deviation to be an accurate measure of risk, an assumption has to be made that investment performance data follows the normal distribution. In graphical terms, a normal distribution of data will plot on a chart in a manner that looks like a bell shaped curve. If this standard holds true, we know then approximately 68% of the expected outcomes should lie between ±1 standard deviations from the investment’s expected return, 95% should lie between ±2 standard deviations, and 99% should lie between ±3 standard deviations. Three main reasons why investment performance data may not be normally distributed.

Firstly, investment performance is typically skewed, which means that return

\(^6\)A Simplified Approach To Calculating Volatility http://www.investopedia.com/articles/basics/09/simplified-measuring-interpreting-volatility
2.3 VaR

distributions are typically asymmetrical. As a result, investors tend to experience abnormally high and low periods of performance.

Second, investment performance typically exhibits something called kurtosis, which means that investment performance has an abnormally large number of positive and/or negative periods of performance. Having together, these problems warp the look of the bell-shaped curve, and distort the accuracy of standard deviation as a measure of risk.

In addition to skewness and kurtosis, last but not least the problem known as heteroskedasticity is also a cause for concern. Heteroskedasticity simply means that the variance of the sample investment performance data is not constant over time. As a result, standard deviation tends to fluctuate based on the length of the time period used to make the calculation, or the period of time selected to make the calculation.

Like skewness and kurtosis, the ramifications of heteroskedasticity will cause standard deviation to be an unreliable measure of risk. Taken collectively, these three problems can cause investors to misunderstand the potential volatility of their investments, and cause them to potentially take much more risk than anticipated.

2.3 VaR

What is the most I can lose on this investment? This is a question that almost every investor who has invested or is considering investing in a risky asset asks at some point in time. So VaR is "What loss level is such that we are X% confident it will not be exceeded in N business days?" or Value at Risk (VaR) is a statistical technique used to measure and quantify the level of financial risk within a firm or investment portfolio over a specific time frame. This metric is most commonly used by investment and commercial banks to determine the extent and occurrence ratio of potential losses in their institutional portfolios. VaR calculations can be applied to specific positions or portfolios as a whole or to measure firm-wide risk.
2.3 VaR

exposure.

Let’s see it more formally call $W$ the value of an asset or a portfolio of assets and $V = W_{t_1} - W_{t_0}$ the random change (revenue) of this value during the period $h = \Delta t = t_1 - t_0$, then VaR is defined as follows:

$$VaR = E(V) - V^* \quad (2.2)$$

$E(V)$ is the expectation of $V$ and the critical revenue $V^*$ is defined by:

$$\int_{-\infty}^{V^*} f(\nu) d\nu = Prob(\nu < V^*) = p. \quad (2.3)$$

Using that $V = W_{t_0} \cdot X$ with $X = ln(W_{t_1}/W_{t_0})$ VaR can also be expressed in terms of the

$$VaR = W_{t_0}(E(X) - X^*) \quad (2.4)$$

where $E(X)$ and $X^*$ are defined analogous to $E(V)$ and $V^*$. Figure (2.2) illustrates the concept graphically. From equation (2.3) and figure (2.2), it is obvious that the calculation of VaR boils down to finding the $p$-quantile of the random variable $V$ (i.e. the profit and loss distribution).

VaR summarizes the: expected maximum loss (or worst loss) over a target horizon within a given confidence level, where extremely adverse scenarios are excluded. Thus, 1-day value-at-risk at 95% confidence level, is: The maximum loss that will occur tomorrow, if the worst 5% situations are not considered or the minimum loss that will occur tomorrow, if only the worst 5% situations are considered.

Value at Risk is an estimate of the worst possible loss an investment could realize over a given time horizon, under normal market conditions (defined by a given level of confidence). Normal market conditions – the returns that account for 95% of the distribution of possible outcomes. Abnormal market conditions – the
returns that account for the other 5% of the possible outcomes. Also Value-at-risk (VaR) according to Holton\(^7\) is a probabilistic metric of market risk (PMMR) used by banks and other organizations to monitor risk in their trading portfolios. For a given probability and a given time horizon, value-at-risk indicates an amount of money such that there is that probability of the portfolio not losing more than that amount of money over that horizon. For example, if a portfolio has a one-day 90% value-at-risk of €3.2 million, such a portfolio would be expected to not lose more than €3.2 million, nine days out of ten.

To define a value-at-risk metric, we must identify three things:

- The period of time over which a possible loss will be calculated—1 day, 2 weeks, 1 month, etc. This is called the value-at-risk horizon. For a bank trading portfolio invested in highly liquid currencies, a one-day horizon may be acceptable. For an investment manager with a monthly rebalancing and reporting focus, a 30-day period may be more appropriate. Ideally, the holding period should correspond to the longest period needed for an orderly portfolio liquidation.

---

\(^7\)https://www.glynholton.com/notes/value-at-risk/
2.3 VaR

- A quantile of that possible loss that defines confidence level. The term “percentile” may be used in place of “quantile”. A percentile is a quantile expressed as a percentage. For example, a 95th percentile is a .95 quantile. If the portfolio’s value-at-risk is expressed as a .90 quantile of loss there is 90% of loss. The choice of the confidence level also depends on its use. If the resulting VARs are directly used for the choice of a capital cushion, then the choice of the confidence level is crucial, as it should reflect the degree of risk aversion of the company and the cost of a loss of exceeding VAR. Higher risk aversion, or greater costs, implies that a greater amount of capital should cover possible losses, thus leading to a higher confidence level. In contrast, if VAR numbers are just used to provide a company-wide yardstick to compare risks across different markets, then the choice of the confidence level is not too important.

- The currency in which the possible loss is denominated. This is called the base currency. If a British bank calculates value-at-risk as the 0.99 quantile of loss over ten trading days, as required under the Basel Accords, this would be called 10-day 99% GBP VaR thus if National Bank Of Greece calculates value-at-risk as the 0.99 quantile of loss over ten trading days this would be called 10-day 99% €VaR

2.3.1 VaR Methods

There are three primary methods used for calculating Value at Risk (VaR).


- b. Historical simulation Non parametric method.

- c. Monte Carlo simulation method.

There are also Semi Parametric methods, for them we just make a small reference, and most of our talk will be about Extreme Value Theory (EVT).
2.3 VaR

All VaR methods have a common base but then diverge in how they actually calculate the number of Value at Risk (VaR). They also have a common problem in assuming that the future will follow the past. This shortcoming is normally addressed by supplementing any VaR figures with appropriate sensitivity analysis and/or stress testing. In general the VaR calculation follows five steps:

- Identification of positions for Value at Risk
- Identification of risk factors affecting valuation of positions.
- Assignment of probabilities (or statistical distribution) to possible risk factors values.
- Creation of pricing functions for positions as a function of values of risk factors.
- Calculation of Value at Risk (VaR)

**Variance-Covariance parametric method**

This VaR calculating method assumes that the daily returns follow a distribution. Let’s assume the Normal distribution. From the distribution of daily returns calculated from daily price series we estimate the standard deviation. The daily Value at Risk VaR is simply a function of the standard deviation and the desired confidence level. In the Variance-Covariance VaR method the underlying volatility may be calculated either using a simple moving average or an exponentially weighted moving average.

Mathematically, the difference lies in the method used to calculate the standard deviation.

The contribution of the RiskMetrics service offered by J.P. Morgan in 1995 was that it made the variances in and covariances across asset classes freely available to anyone who wanted to access them, thus easing the task for anyone who wanted to compute the Value at Risk analytically for a portfolio. J.P. Morgan in 1996

This approach is generally utilized if it is believed that the daily returns during the look back period have followed normal market conditions. The simple moving average approach places equal importance to all returns in the series whereas the approach of exponentially weighted moving average places greater emphasis on returns of more recent duration.

But for the Variance-Covariance method’s assumptions also pros and cons we will make more references later to the chapter of models as we will specialize our study to the more advanced applications of this method.

**Historical simulation -Non parametric method**

Historical simulations represent the simplest way of estimating the Value at Risk for many portfolios. The historical method simply re-organizes actual historical returns, putting them in order from worst to best. It then assumes that history will repeat itself, from a risk perspective. Because the lowest 5% of daily returns (the returns are ordered from left to right of from worst to best, the worst are always the 'left tail') are returns for example starting from -4% to -8% even more, these are the worst 5% of all daily returns, we can say with 95% confidence that the worst daily loss will not exceed 4% or we expect with 95% confidence that our gain will exceed -4%. If we invest 100 €, we are 95% confident that our worst daily loss will not exceed 4 € (100 x -4%). The interpretation of VaR statistic into both percentage and € terms is the same in Parametric and non Parametric method.

While historical simulations are popular and relatively easy to run, also saves us the trouble and related problems of having to make specific assumptions about distributions of returns. But they do come with baggage. In particular, the underlying assumptions of the model generate give rise to its weaknesses.

-The limitation of the historical simulation lies in its i.i.d. assumption of returns: From empirical evidence, it is known that asset returns are clearly not in-
dependent as they exhibit certain patterns such as volatility clustering. Unfortunately Historical Simulation does not take into account such patterns.

-Past is not prologue: While all three approaches to estimating VaR use historical data, historical simulations are much more reliant on them than the other two approaches for the simple reason that the Value at Risk is computed entirely from historical price changes. There is little room to overlay distributional assumptions (as we do with the Variance-covariance approach) or to bring in subjective information (as we can with Monte Carlo simulations).

-Trends in the data: A related argument can be made about the way in which we compute Value at Risk, using historical data, where all data points are weighted equally. In other words, the price changes from trading days in 1992 affect the VaR in exactly the same proportion as price changes from trading days in 1998. To the extent that there is a trend of increasing volatility even within the historical time period, we will understate the Value at Risk.

-New assets or market risks: While this could be a critique of any of the three approaches for estimating VaR, the historical simulation approach has the most difficulty dealing with new risks and assets for an obvious reason: there is no historic data available to compute the Value at Risk. Assessing the Value at Risk to a firm from developments in online commerce in the late 1990s would have been difficult to do, since the online business was in its nascent stage.

In a market where risks are volatile and structural shifts occur at regular intervals the above assumptions are difficult to sustain.

As with the other approaches to computing VaR, there have been modifications and evolution suggested to the approach, largely directed at taking into account some of the criticisms mentioned in the last section namely some are:

-Weighting the recent past more, in simple terms, each return, rather than being weighted equally, is assigned a probability weight based on its recency. In other words, if the decay factor is .90, the most recent observation has the probability weight p, the observation prior to it will be weighted 0.9p, the one before that is
2.3 VaR

weighted 0.81p and so on.

-Combining historical simulation with time series models: Cabedo and Moya\textsuperscript{9} suggested that better estimates of VaR could be obtained by fitting at time series model through the historical data and using the parameters of that model to forecast the Value at Risk. In particular, they fit an autoregressive moving average (ARMA) model to the oil price data from 1992 to 1998 and use this model to forecast returns with a 99\% confidence interval for the holdout period of 1999. The actual oil price returns in 1999 fall within the predicted bounds 98.8\% of the time, in contrast to the 97.7\% of the time that they do with the unadjusted historical simulation. One big reason for the improvement is that the measured VaR is much more sensitive to changes in the variance of oil prices with time series models, than with the historical simulation -Volatility Updating: Hull and White\textsuperscript{10} suggest a different way of updating historical data for shifts in volatility. For assets where the recent volatility is higher than historical volatility, they recommend that the historical data be adjusted to reflect the change.(it’s kind of a semi parametric method)

Monte Carlo simulation method

In this section, we take a brief look to the use of Monte Carlo simulations as a risk assessment tool. These simulations happen to be useful in assessing Value at Risk, with the focus on the probabilities of losses exceeding a specified value rather than on the entire distribution. Monte Carlo method was coined in the 1940s by John Von Neumann, Stanislaw Ulam and Nicholas Metropolis. Monte Carlo simulation uses random samples from known populations of simulated data to track a statistic’s behavior. With Monte Carlo VaR measures, an inference procedure typically characterizes the distribution of returns by assuming some standard joint distribution—such as the joint-normal distribution—and specifying a covariance ma-


trix and mean vector.

The first two steps in a Monte Carlo simulation mirror the first two steps in the Variance-Covariance method where we identify the market risks that affect the asset or assets and convert the individual assets in a portfolio into positions in standardized instruments. It is in the third step that the differences emerge also with Historical method. Rather than compute the variances and covariances across the market risk factors, we take the simulation route, where we specify probability distributions for each of the market risk factors and specify how these market risk factors move together.

While the estimation of parameters is easier if you assume normal distributions for all variables, the power of Monte Carlo simulations comes from the freedom you have to pick alternate distributions for the variables. In addition, you can bring in subjective judgments to modify these distributions. Once the distributions are specified, the simulation process starts. In each run, the market risk variables take on different outcomes and the value of the portfolio reflects the outcomes. After a repeated series of runs, numbering usually in the thousands, you will have a distribution of portfolio values that can be used to assess Value at Risk. For instance, assume that you run a series of 10,000 simulations and derive corresponding values for the portfolio. These values can be ranked from highest to lowest, and the 95th percentile Value at Risk will correspond to the 500th lowest value and the 99th percentile to the 100th lowest value.

A simulation is only as good as the probability distribution for the inputs that are fed into it. While Monte Carlo simulations are often touted as more sophisticated than historical simulations, many users directly draw on historical data to make their distributional assumptions. In addition, as the number of market risk factors increases and their co-movements become more complex, Monte Carlo simulations become more difficult to run for two reasons. First, you now have to estimate the probability distributions for hundreds of market risk variables rather than just the handful that we talked about in the context of analyzing a single
project or asset. Second, the number of simulations that you need to run to obtain reasonable estimate of Value at Risk will have to increase substantially (to the tens of thousands from the thousands). The strengths of Monte Carlo simulations can be seen when compared to the other two approaches for computing Value at Risk. Unlike the variance-covariance approach, we do not have to make unrealistic assumptions about normality in returns. In contrast to the historical simulation approach, we begin with historical data but are free to bring in both subjective judgments and other information to improve forecasted probability distributions. Finally, Monte Carlo simulations can be used to assess the Value at Risk for any type of portfolio and are flexible enough to cover options and option-like securities.

Also with the other approaches to computing VaR, there have been modifications and evolution suggested to the approach:

- Scenario Simulation: One way to reduce the computation burden of running Monte Carlo simulations is to do the analysis over a number of discrete scenarios.
- Monte Carlo Simulations with Variance-Covariance method modification: The strength of the Variance-covariance method is its speed. If you are willing to make the required distributional assumption about normality in returns and have the variance-covariance matrix in hand, you can compute the Value at Risk for any portfolio in minutes. The strength of the Monte Carlo simulation approach is the flexibility it offers users to make different distributional assumptions and deal with various types of risk, but it can be painfully slow to run.

**Semi Parametric method Extreme Value Theory**

Standard VaR methods, such as variance-covariance method or historical simulation, can fail when the return distribution is fat tailed. This problem is aggravated when long-term VaR forecasts are desired. The semi-parametric approaches of Hull and White we saw before, Extreme Value Theory (EVT), etc are some of these methods.
ETV is proposed to overcome these problems. EVT can be considered as a state-of-the-art procedure for estimating the downside risk of a distribution. However, it has now been applied in the financial studies more than before. If the prediction of very rare events is desired and leptokurtic distributions are involved. Now we turn to the Extreme Value Theory (EVT) in order to improve the estimation of extreme quantiles. EVT provides statistical tools to estimate the tails of probability distributions.

2.3.2 Limitations of VaR

While Value at Risk has acquired a strong following in the risk management community, there is reason to be skeptical of both its accuracy as a risk management tool and its use in decision making. There are many dimensions on which researcher have taken issue with VaR and we will categorize the criticism into those dimensions. *VaR can be wrong*

There is no precise measure of Value at Risk, and each measure comes with its own limitations. The end-result is that the Value at Risk that we compute for an asset, portfolio or a firm can be wrong, and sometimes, the errors can be large enough to make VaR a misleading measure of risk exposure. The reasons for the errors can vary across firms and for different measures and include the following.

- **Return distributions:** Every VaR measure makes assumptions about return distributions, which, if violated, result in incorrect estimates of the Value at Risk.  
- **History may not be a good predictor:** All measures of Value at Risk use historical data to some degree or the other.  
- **Non-stationary Correlations:** Measures of Value at Risk are conditioned on explicit estimates of correlation across risk sources (in the variance-covariance and Monte Carlo simulations) or implicit assumptions about correlation (in historical simulations). These correlation estimates are usually based upon historical data and are extremely volatile.
Chapter 3

PARAMETRIC MODELS

If a parametric approach to VaR estimation is utilized, the question arises which distribution function fits best to the observed changes of the market factors. For example J.P. Morgan’s Risk Metrics in 1996 describe the assumptions underlying their computation of VaR:

a) Returns on individual risk factors are assumed to follow conditional normal distributions. While returns themselves may not be normally distributed and large outliers are far too common (i.e., the distributions have fat tails), the assumption is that the standardized return (computed as the return divided by the forecasted standard deviation) is normally distributed.

b) The focus on standardized returns implies that it is not the size of the return per se that we should focus on but its size relative to the standard deviation. In other words, a large return (positive or negative) in a period of high volatility may result in a low standardized return, whereas the same return following a period of low volatility will yield an abnormally high standardized return.

The focus on normalized standardized returns exposed the VaR computation to the risk of more frequent large outliers than would be expected with a normal distribution. In a subsequent variation, the RiskMetrics approach was extended to cover normal mixture distributions, which allow for the assignment of higher probabilities for outliers. In effect, these distributions require estimates of the prob-
abilities of outsized returns occurring and the expected size and standard deviations of such returns, in addition to the standard normal distribution parameters. Even proponents of these models concede that estimating the parameters for jump processes, given how infrequently jumps occur, is difficult to do.

So the strength of the Variance-Covariance approach is that the Value at Risk is simple to compute, once you have made an assumption about the distribution of returns and inputted the means, variances and covariances of returns. In the estimation process, though, lie the three key weaknesses of the approach:

- Wrong distributional assumption: If conditional returns are not normally distributed, the computed VaR will understate the true VaR. In other words, if there are far more outliers in the actual return distribution than would be expected given the normality assumption, the actual Value at Risk will be much higher than the computed Value at Risk.

- Input error: Even if the standardized return distribution assumption holds up, the VaR can still be wrong if the variances and covariances that are used to estimate it are incorrect. To the extent that these numbers are estimated using historical data, there is a standard error associated with each of the estimates. In other words, the variance-covariance matrix that is input to the VaR measure is a collection of estimates, some of which have very large error terms.

- Non-stationary variables: A related problem occurs when the variances and covariances across assets change over time. This nonstationarity in values is not uncommon because the fundamentals driving these numbers do change over time. Thus, the correlation between the U.S. Dollar and the Japanese Yen may change if oil prices increase by 15%. This, in turn, can lead to a breakdown in the computed VaR.

Not surprisingly, much of the work that has been done to revitalize the approach has been directed at dealing with these critiques. First researchers have examined how best to compute VaR with assumptions other than the standardized normal.
Hull and White\textsuperscript{1} suggest ways of estimating Value at Risk when variables are not normally distributed. This and other papers like it develop interesting variations but have to overcome two practical problems.

Estimating inputs for non-normal models can be very difficult to do, especially when working with historical data, and the probabilities of losses and Value at Risk are simplest to compute with the normal distribution and get progressively more difficult with asymmetric and fat-tailed distributions.

Second, other research has been directed at making better the estimation techniques to yield more reliable variance and covariance values to use in the VaR calculations.

Some suggest refinements on sampling methods and data innovations that allow for better estimates of variances and covariances looking forward. Others posit that statistical innovations can yield better estimates from existing data.

For instance, conventional estimates of VaR are based upon the assumption that the standard deviation in returns does not change over time (homoskedasticity), Engle argues that we get much better estimates by using models that explicitly allow the standard deviation to change over time (heteroskedasticity)\textsuperscript{2} In fact, he suggests two variants – Autoregressive Conditional Heteroskedasticity (ARCH) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) – that provide better forecasts of variance and, by extension, better measures of Value at Risk.\textsuperscript{3}

One final critique that can be leveled against the variance-covariance estimate of VaR is that it is designed for portfolios where there is a linear relationship between risk and portfolio positions.

\textsuperscript{1}Hull, J. and A. White, 1998, Value at Risk when daily changes are not normally distributed, Journal of Derivatives, v5, 9-19.


\textsuperscript{3}He uses the example of a $1,000,000 portfolio composed of 50% NASDAQ stocks, 30% Dow Jones stocks and 20% long bonds, with statistics computed from March 23, 1990 to March 23, 2000. Using the conventional measure of daily standard deviation of 0.83% computed over a 10-year period, he estimates the value at risk in a day to be $22,477. Using an ARCH model, the forecast standard deviation is 1.46%, leading to VaR of $33,977. Allowing for the fat tails in the distribution increases the VaR to $39,996.
Consequently, it can break down when the portfolio includes options, since
the payoffs on an option are not linear. In an attempt to deal with options and
other non-linear instruments in portfolios, researchers have developed Quadratic
Value at Risk measures. These quadratic measures, sometimes categorized as delta-
gamma models (to contrast with the more conventional linear models which are
called delta-normal), allow researchers to estimate the Value at Risk for compli-
cated portfolios that include options and option-like securities such as convertible
bonds. The cost, though, is that the mathematics associated with deriving the VaR
becomes much complicated and that some of the intuition will be lost along the
way.

Some important questions to first consider when first looking at a time
series so then to choose a model

- Is there a trend, meaning that, on average, the measurements tend to in-
crease (or decrease) over time?

- Is there seasonality, meaning that there is a regularly repeating pattern of
highs and lows related to calendar time such as seasons, quarters, months,
days of the week, and so on?

- Are their outliers? In regression, outliers are far away from your line. With
time series data, your outliers are far away from your other data.

- Is there a long-run cycle or period unrelated to seasonality factors?

- Is there constant variance over time, or is the variance non-constant?

- Are there any abrupt changes to either the level of the series or the variance?

- Are features - evidence of the above by looking at the plot?

- What is that we want to study e.g. what’s my forecast for the variable itself
next period? (ARIMA) Or given the past data, what’s my forecast of the
shock volatility next period? (ARCH GARCH) This is very important and it
is mainly the difference between the tools we use for studying timeseries. So in our case and if I want to predict the VaR of a timeseries return, I need to model this using something from GARCH model family, since VaR is relied upon the volatility of the asset, and not the mean returns that ARIMA would model for!

3.0.1 ARIMA

Continuing our trip we examine first the ARIMA models, also called Box-Jenkins models, those are models that may possibly include autoregressive terms, moving average terms, and differencing operations. Various abbreviations are used: When a model only involves autoregressive terms it may be referred to as an AR model. When a model only involves moving average terms, it may be referred to as an MA model. When no differencing is involved, the abbreviation ARMA may be used.

The two common concepts of conditional mean the AR process and the MA process.

The AR process is described by the Equation 3.1:

\[ y_t = \mu + \sum_{i=1}^{p} \rho_i * y_{t-1} + \epsilon_t \]  

(3.1)

where \( p \) is the lag parameter of the observed variable, \( y_t \) is the random observed variable at time \( t \) depending on the previously realized values of \( y_{t-1} \), \( \mu \) is the mean constant and \( \epsilon_t \) is the white noise.

The MA process is described by the Equation 3.2:

\[ y_t = \mu + \sum_{i=1}^{q} \theta_i * \epsilon_{t-1} + \epsilon_t \]  

(3.2)

where \( q \) is the number of lags of the error term, \( y_t \) is the random observed variable depending on the previously realized values of error term \( \epsilon_{t-1} \), \( \theta \) is the parameter, \( \mu \) is the mean constant, \( \epsilon_t \) is the white noise.
The combination of both gives us the ARMA process described by the Equation 3.3

\[ y_t = \mu + \sum_{i=1}^{p} \rho_i * y_{t-1} + \sum_{i=1}^{q} \theta_i * \epsilon_{t-1} + \epsilon \]  \hspace{1cm} (3.3)

Autoregressive Integrated Moving Average (ARIMA) model is a generalization of an Autoregressive Moving Average (ARMA) model. Both of these models are fitted to time series data either to better understand the data or to predict future points in the series (forecasting). ARIMA models are applied in some cases where data show evidence of non-stationarity, where an initial differencing step (corresponding to the 'integrated' part of the model) can be applied one or more times to eliminate the non-stationarity.

As the financial data time series shows heteroskedasticity a model dealing with conditional heteroskedasticity must be used, a model that specifies the conditional variance of the process.

We will see at once the use the GARCH model introduced by (Bollerslev, 1986), which is a generalization of the ARCH model that was originally developed by (Engle, 1982).

### 3.0.2 ARCH GARCH

The great workhorse of applied econometrics is the least squares model. This is natural because applied econometricians are typically called upon to determine how much one variable will change in response to a change in some other variable. Increasingly however, econometricians are being asked to forecast and analyze the size of the errors of the model. In this case the questions are about volatility and the standard tools have become the ARCH GARCH models. The basic version of the least squares model assumes that, the expected value of all error terms when squared, is the same at any given point. This assumption is called homoskedasticity and it is this assumption that is the focus of ARCH GARCH models. Data in which the variances of the error terms are not equal, in which the error terms may
reasonably be expected to be larger for some points or ranges of the data than for
others, are said to suffer from heteroskedasticity$^4$.

Engle continues and says, the standard warning is that in the presence of het-
eroskedasticity, the regression coefficients for an ordinary least squares regression
are still unbiased, but the standard errors and confidence intervals estimated by
conventional procedures will be too narrow, giving a false sense of precision.Instead
of considering this as a problem to be corrected, ARCH and GARCH models
treat heteroskedasticity as a variance to be modeled. As a result, not only are
the deficiencies of least squares corrected, but a prediction is computed for the
variance of each error term. This turns out often to be of interest particularly in
finance. The ARCH and GARCH models, which stand for autoregressive condi-
tional heteroskedasticity and generalized autoregressive conditional heteroskedas-
ticity, are designed to deal with volatility clustering $^5$ issues.

**Autoregressive (AR):** tomorrow’s variance (or volatility) is a regressed function of
today’s variance, it regresses on itself.

**Conditional (C):** tomorrow’s variance depends, is conditional on, the most recent
variance. An unconditional variance would not depend on today’s variance.

**Heteroskedastic (H):** variances are not constant, they flux over time GARCH re-
gresses on “lagged” or historical terms. The lagged terms are either variance or
squared returns. The generic GARCH (p, q) model regresses on (p) squared returns
and (q) variances. Therefore, GARCH (1, 1) “lags” or regresses on last period’s
squared return (i.e., just 1 return) and last period’s variance (i.e., just 1 vari-
ance). Those models have become widespread tools for dealing with time series
heteroskedastic models. The goal of such models is to provide a volatility measure,
like a standard deviation, that can be used in financial decisions concerning risk
analysis, portfolio selection and derivative pricing. Although an ARCH model

---

157-168

$^5$In finance, volatility clustering refers to the observation, as noted as Mandelbrot (1963),
that large changes tend to be followed by large changes, of either sign, and small changes tend
to be followed by small changes.
could possibly be used to describe a gradually increasing variance over time, most often it is used in situations in which there may be short periods of increased variation. (Gradually increasing variance connected to a gradually increasing mean level might be better handled by transforming the variable.)

The ARCH model allows for long lags in conditional variance and the GARCH model extends it in the way that it allows for both long lags in conditional variance and a more flexible lag structure.

The definition of the GARCH\((p,q)\) model is described by the equation \(3.4\)

\[
\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2
\]  

where \(p\) is the order of the GARCH terms \(\sigma_t^2\) and \(q\) is the order of the ARCH terms \(\epsilon_t^2\).

**Generalizations**

An ARCH\((m)\) process is one for which the variance at time \(t\) is conditional on observations at the previous \(m\) times, and the relationship is:

\[
\text{Var}(y_t | y_{t-1}, \ldots, y_{t-m}) = \sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \cdots + \alpha_m y_{t-m}^2.
\]

With certain constraints imposed on the coefficients, the \(y_t\) series squared will theoretically be AR\((m)\).

A GARCH (generalized autoregressive conditionally heteroscedastic) model uses values of the past squared observations and past variances to model the variance at time \(t\). As an example, a GARCH\((1,1)\) is:

\[
\sigma_t^2 = \alpha_0 + a_1 y_{t-1}^2 + \beta_1 \sigma_{t-1}^2
\]  

In the GARCH notation, the first subscript refers to the order of the \(y^2\) terms on the right side, and the second subscript refers to the order of the \(\sigma^2\) terms in other words the GARCH \((1, 1)\) model solves for the conditional variance as a function of three variables previous variance, previous square return, and long-run
For identifying an ARCH GARCH Model in practice the best identification tool may be a time series plot of the series. It’s usually easy to spot periods of increased variation sprinkled through the series. It can be fruitful to look at the ACF and PACF of both $y_t$ and $y_t^2$.

For instance, if $y_t$ appears to be white noise and $y_t^2$ appears to be AR(1), then an ARCH(1) model for the variance is suggested. If the PACF of the $y_t^2$ suggests AR(m), then ARCH(m) may work. GARCH models may be suggested by an ARMA type look to the ACF and PACF of $y_t^2$. In practice, things won’t always fall into place as nicely as we might expect. We might have to experiment with various ARCH and GARCH structures after spotting the need in the time series plot of the series.

The following plot is a time series plot of a simulated series, $x$, ($n = 300$) for the GARCH(1,1) model of:

$$\text{Var}(x_t|x_{t-1}) = \sigma^2_t = 5 + 0.5x_{t-1}^2 + 0.5\sigma^2_{t-1}.$$ 

**Figure 3.1: Garch(1,1)**

The ACF of the series below shows that the series looks to be white noise. Note that correlations are not significant.

The ACF of the squared series follows an ARMA pattern because both the
Figure 3.2: ACF PACF X

ACF and PACF taper. This suggests a GARCH(1,1) model.

Figure 3.3: ACF PACF X^2
3.0.3 LONG MEMORY IN VOLATILITY ARFIMA and FIGARCH

How persistent is volatility? In other words, how quickly do financial markets forget large volatility shocks? Shephard\textsuperscript{6} shows that daily squared returns on exchange rates and stock indices can have autocorrelations which are significant for many lags. In any stationary ARCH or GARCH model, memory decays exponentially fast\textsuperscript{7} also any pure ARIMA stationary time series can be considered a short memory series. An AR(p) model has infinite memory, as all past values of $\varepsilon_t$ are embedded in $y_t$, but the effect of past values of the disturbance process follows a geometric lag, damping off to near-zero values quickly. A MA(q) model has a memory of exactly q periods, so that the effect of the moving average component quickly dies off and as we saw before In estimating an ARIMA model, the researcher chooses the integer order of differencing $d$ to ensure that the resulting series $(1 - L)^d y_t$ is a stationary process. As unit root tests often lack the power to distinguish between a truly nonstationary series and a stationary series embodying a structural break or shift, time series are often first-differenced if they do not receive a clean bill of health from unit root testing. Many time series exhibit too much long-range dependence to be classified as stationary but are not stationary. The ARFIMA model is designed to represent these series. Also when financial return series exhibit long memory behavior in volatility, the GARCH models, which only capture the short-run dependencies, could have poor performance (see Baille et al., 1996 and 2000). Several models were proposed to incorporate the long memory property of volatility in financial time series in recent years. To allow for


\textsuperscript{7}In the formula 3.5, persistence is $= (\alpha-1+ \beta-1)$. Persistence refers to how quickly (or slowly) the variance reverts or “decays” toward its long-run average. High persistence equates to slow decay and slow “regression toward the mean;” low persistence equates to rapid decay and quick “reversion to the mean.” A persistence of 1.0 implies no mean reversion. A persistence of less than 1.0 implies “reversion to the mean,” where a lower persistence implies greater reversion to the mean.
fractional integrated processes of the conditional variance and therefore, provide
a useful model for series in which the conditional variance is persistent, Baillie et
al. (1996) proposed the fractionally integrated generalized autoregressive condi-
tional heteroscedasticity (FIGARCH) model by generalizing the IGARCH model
to allow for persistence in the conditional variance.

But first lets see how we get there, one of the most popular long memory
model for levels \( x_t \) is the ARFIMA \((p,d,q)\) due to Hosking (1981) and Granger
and Joyeux (1980). The FI in ARFIMA stands for 'Fractionally Integrated'. In
other words, ARFIMA models are simply ARIMA models in which the \( d \) (the
degree of integration) is allowed to be a fraction of a whole number, such as 0.4,
instead of an integer, such as 0 or 1. The idea of a fractional difference may seem
puzzling at first. It is easy to take the \( d \)th difference when \( d \) is 0 1 or 2, but what
if \( d = 0.4 \)? The ARFIMA \((p, d, q)\), with mean \( \mu \), may be written using operator
notation as

\[
\Phi(L)(1-L)^d(y_t - \mu) = \Theta(L)\epsilon_t, \epsilon \sim i.i.d.(0, \sigma^2)
\]  

(3.6)

where \( L \) is the backward-shift operator

\[
\Phi(L) = 1 - \phi_1L - \ldots - \phi_pL^p
\]  

(3.7)

also

\[
\Theta(L) = 1 + \theta_1L + \ldots + \theta_qL^2
\]  

(3.8)

and \((1-L)^d\) is the fractional differencing operator defined by:

\[
(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)L^k}{\Gamma(-d)\Gamma(k+1)}
\]  

(3.9)

with \( \Gamma \) denoting the gamma (generalized factorial) function. The parameter \( d \)
is allowed to assume any real value. The arbitrary restriction of \( d \) to integer values
gives rise to the standard autoregressive integrated moving average (ARIMA)
model. The stochastic process \( y_t \) is both stationary and invertible if all roots of \( \Phi(L) \)
and $\Theta (L)$ lie outside the unit circle and $|d| < 0.5$. The process is nonstationary for $d \geq 0.5$, as it possesses infinite variance\(^8\). So ARFIMA drives us to FIGARCH.

The new class of Fractionally Integrated Generalized AutoRegressive Conditionally Heteroskedastic (FIGARCH) processes was introduced by Baillie\(^9\). The primary purpose of this approach was to develop a more flexible class of processes for the conditional variance that are more capable of explaining and representing the observed temporal dependencies in financial markets volatility. The FIGARCH process combines many of the features of the fractionally integrated process for the mean together with the regular GARCH process for the conditional variance. In particular, the FIGARCH model implies a slow hyperbolic rate of decay for the lagged squared innovations in the conditional variance function, although the cumulative impulse response weights associated with the influence of a volatility shock on the optimal forecasts of the future conditional variance eventually tend to zero.

In equation 3.4 the primary constraint of this model is that all the expounding variables must be positive i.e. $\alpha_0, \alpha, \beta \geq 0$ this is known as the non-negativity restriction. Further, for stationarity we require that $\alpha + \beta$ is less than unity. However, if this restriction violates, i.e. $\alpha + \beta \geq 1$ we conclude that the shocks are persistent. Hence, to account for the persistency of shocks an IGARCH (1,1) model proposed by Engle and Bollerslev (1986) can be rewritten as:

$$h_t = w + \sum_{i=1}^{q} (1 - \beta_i) \epsilon_{t-1}^2 + \sum_{i=1}^{q} \beta_i h_{t-i}$$

(3.10)

where $h_t = \sigma^2$, $w$ is constant and $0 \leq \beta \leq 1$.

The IGARCH model implies infinite persistence of the conditional variance to a shock in squared returns. The IGARCH process can also be illustrated as an

---

\(^8\)text Granger and Joyeux (JTSA, 1980)

ARMA (m,p) process:

\[ \Phi(L)(1 - L)\varepsilon_t^2 = w + [1 - \beta(L)]\nu_t \]  \hspace{1cm} (3.11)

The \( \nu_t \) process can be interpreted as the “innovations” for the conditional variance, as it is a zero-mean martingale.\(^{10}\) The fractionally integrated GARCH or FIGARCH class of models is obtained by replacing the first difference operator \((1 - L)\) in the above model with the fractional differencing operator \((1 - L)^d\) where \(d\) is a fraction \(0 < d < 1\). Thus, the FIGARCH class of models can be obtained by considering:

\[ \Phi(L)(1 - L)^d\varepsilon_t^2 = w + [1 - \beta(L)]\nu_t \]  \hspace{1cm} (3.12)

where, \(L\) denotes the lag or backshift operator. \(\varepsilon_t\) are serially uncorrelated errors having zero mean. \(\varepsilon_t^2\) are squared errors of GARCH process.

The process of \(\nu_t\) is integrated for conditional variance \(\sigma_t^2\) as variations. \(\nu_t = \varepsilon_t^2 - \sigma_t^2\).

It is assumed that all roots of \(\Phi(L)\) and \([1 - \beta(L)]\) stayed out of unit circle. The parameter \(d\) is the fractional integration parameter showing the degree of long memory or persistence of shocks to conditional variance. \(d\) parameter satisfies the condition \(0 \leq d \leq 1\). If \(0 < d < 1\), the model indicates an intermediate range of long memory.

It means that volatility shocks die hyperbolically.

If \(d = 0\), then the process of FIGARCH (p,d,q) is reduced to the process of GARCH (p,q).

If \(d = 1\), then the process of FIGARCH becomes an integrated process of GARCH (IGARCH).

The Model 3.12 can be reformed as follows (Baillie et al., 1996):

\[ [1 - \beta(L)]\sigma_t^2 = w + [1 - \beta(L)(1 - L)^d]\nu_t\varepsilon_t^2 \]  \hspace{1cm} (3.13)

\(^{10}\)An unbiased random walk (in any number of dimensions) is an example of a martingale.
where $\sigma^2_t$, which is conditional variance of $\varepsilon^2_t$, is displayed with

$$
\sigma^2_t = \frac{\omega}{[1 - \beta(L)]} + \lambda(L) \varepsilon^2_t
$$

(3.14)

and where

$$
\lambda(L) = 1 - \frac{\phi(L)}{[1 - \beta(L)]} + (1 - L)^d
$$

(3.15)

The FIGARCH process identifies potential presence of long memory or the subsistence of dependencies in financial time series mainly due to the hyperbolically decaying autocorrelation function as we already mention, or in other words, long memory process can be illustrated through a fractionally integrated procedure. This means, the level of integration is less than one, but however superior to zero, implying that the impacts of a shock continue over an extensive period of time. The main advantage of the FIGARCH process is that it allows for long memory in the conditional variance which is characterized by the fractional integration parameter $d$ and the short-term dynamics can be modeled through the traditional GARCH parameters.
Chapter 4

EMPIRICAL FINDINGS

4.1 Preliminary Analysis of Data

This study\(^1\) considers time series data set of Greek Athens Financial Market (ATHEX or ASE Athens Stock Exchange). The data consists of daily closing price, and covers the sample period from 5/2/88 to August 31/7/17 except 28/6/2015 to 3/8/2015. The daily price series are defined as the logarithmic difference of the daily closing index prices. The returns at time \(t\) are obtained by:

\[
RASEIN_t = \ln(P_t/P_{t-1}) \times 100, \quad t = 1, 2, \ldots T
\]  

(4.1)

where \(P_t\) is current index price and \(P_{t-1}\) is the previous day’s index price. RASEIN\(_t\) is the return in percent. The descriptive graphs of the daily closing index price and return series are presented in figures above (a) level of price, (b) returns, (c) histogram and descriptive statistics of RASEIN\(_t\).

Figure 4.1 presents the sample data covering the daily closing prices. From Figure 4.2, it can be said that the conditional variances of returns display volatility clustering which change over time, and they are not independent. In other words, volatility clustering is clearly observable in the graphs. The density graph show

\(^1\)All calculations in this section realised with the help of GRETL econometrics software. http://gretl.sourceforge.net
that returns distributions exhibit fat tails. Table 4.1 summarizes the descriptive statistics of the RASEIN\textsubscript{t} series. As shown in 4.1, the Jarque-Bera statistics rejects the null hypothesis of normality.

In addition, the measures of skewness and kurtosis display that the distribution of returns is leptokurtic and slightly negatively skewed.

![Graph of ATHENS GENERAL STOCK MARKET INDEX 5/2/88 to 31/7/17](image1)

**Figure 4.1:** ATHENS GENERAL STOCK MARKET INDEX 5/2/88 to 31/7/17

![Graph of LOG RETURNS OF ASE INDEX](image2)

**Figure 4.2:** LOG RETURNS OF ASE INDEX

\textsuperscript{2}kurtosis greater than three
4.1 Preliminary Analysis of Data

Summary Statistics, using the observations 1988-02-05 2017-07-31 for the variable RASEIN (7536 valid observations)

<table>
<thead>
<tr>
<th></th>
<th>RASEIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.013473</td>
</tr>
<tr>
<td>Median</td>
<td>0.0000</td>
</tr>
<tr>
<td>Min</td>
<td>-17.713</td>
</tr>
<tr>
<td>Max</td>
<td>14.084</td>
</tr>
<tr>
<td>Std.Dev</td>
<td>1.8597</td>
</tr>
<tr>
<td>C.V.</td>
<td>138.03</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.13209</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>9.6889</td>
</tr>
<tr>
<td>5%perc</td>
<td>-2.8637</td>
</tr>
<tr>
<td>95%perc</td>
<td>2.8350</td>
</tr>
<tr>
<td>IQ RANGE</td>
<td>1.6773</td>
</tr>
<tr>
<td>Jarque-Bera test</td>
<td>14070.7 (00000)</td>
</tr>
<tr>
<td>Missing obs.</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.1. summary statistics of RASEIN

Before examining the volatility and start estimating, we test for stochastic trends in the autoregressive representation of the return series using unit root
The ADF and KPSS tests are used to check whether or not the series is stationary, I(0). Table 4.2 reports the results of both tests. The results indicate that the ADF statistic significantly rejects the null hypothesis of unit roots for daily return series. As for the KPSS test, the test statistics indicate that return series is insignificant to reject the null hypothesis of stationarity, suggesting that they are stationary processes\(^3\) I(0). Hence, the futures daily returns are stationary and suitable for our empirical analysis.

<table>
<thead>
<tr>
<th>Test</th>
<th>Feature</th>
<th>Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF</td>
<td>(\eta_\mu)</td>
<td>-59.8027***</td>
</tr>
<tr>
<td>ADF</td>
<td>(\eta_r)</td>
<td>-59.8522***</td>
</tr>
<tr>
<td>KPSS</td>
<td>(\eta_\mu)</td>
<td>0.631837</td>
</tr>
<tr>
<td>KPSS</td>
<td>(\eta_r)</td>
<td>0.0655984</td>
</tr>
</tbody>
</table>

Note: \(\eta\) refer to the test statistics with and without trend, \(\tau, \mu\) respectively. *denotes significance level at 10%. ** denotes significance level at 5%. *** denotes significance level at 1%.

Table 4.2. Unit Root Tests

---

\(^3\)The KPSS is more powerful than Augmented Dickey-Fuller (ADF) test. The null hypothesis of the ADF test is that a time series contains a unit root, while the KPSS test has the null hypothesis of stationarity. Since the null hypothesis in ADF test is that a time series contains a unit root, this hypothesis is accepted unless there is strong evidence against it. However, this approach may have low power against stationary near unit root processes.
4.2 Long Memory and Asymmetry for RASEIN

In order to examine symmetric\(^4\) long memory property in volatility of return series of Athens Stock Market, FIGARCH models are estimated for different lags (p, q) under assumption of Student-t(ST) and Skewed Student-t(SST) distributions. The different FIGARCH(p,d,q) models as p,q=0,1,2 for RASEIN return series are estimated and compared in terms of Akaike (AIC) and Schwarz (SIC) Information Criteria.\(^5\)

Table 4.3 presents estimation results of most appropriate model FIGARCH(1,d,1) for RASEIN. FIGARCH (1, 1) model is employed in order to investigate the existence of possible temporal dependencies in the volatility of all the sectors of the Greek stock market under investigation. The FIGARCH process identifies potential presence of long memory or the subsistence of dependencies in financial time series mainly due to the hyperbolically decaying autocorrelation function. As per our results, the fractional differencing parameter, d, is found to be significantly different from zero and is within the theoretical value (i.e., 0<d<1).

This indicates that the volatility of all the sectors of the ASE clearly exhibits a long memory process. It is our connotation that our findings show the importance of modeling long memory in volatility and suggests that future volatility depends on its past realizations and, as a result, is predictable. Our findings also support the findings of prior studies on both emerging and developed markets.

The estimated GARCH parameters \(\alpha_1\) and \(\beta_1\) are positive and statistically significant but our concern is more for \(\beta_1\) parameter. We report the sample skewness and kurtosis for the standardized residuals, also Ljung-Box tests for up to 5th-

---

\(^4\)All calculations in this section were realised with TSM4 econometrics software. http://www.timeseriesmodelling.com The problem of finding the right software to manage FIGARCH models was crucial during working this chapter. Software mechanics must apply the ability their products to work with and apply more models as econometrics science evolve. Special thanks to James Davidson for TSM software.

\(^5\)The models with different orders are estimated for both GARCH and FIGARCH under two different distributions. The model selection is based on Akaike’s information criterion (AIC) and Ljung-Box Q-statistics. The model which has the lowest AIC and passes Q-test simultaneously is used.
order serial correlation in the standardized and the squared standardized residuals as diagnostic tests for two models. Moreover, tail parameter “ν” is statistically significant for all of the distributions (ST, SST). While comparing we found FIGARCH(1,d,1) Skewed Student-t(SST) distributions model performs better than the Student-t(ST), which is again consistence with the most recent prior research on the topic.

<table>
<thead>
<tr>
<th>FIGARCH(1,d,1)</th>
<th>Estimation results of RASEIN features</th>
</tr>
</thead>
<tbody>
<tr>
<td>p=1,q=1</td>
<td>Distributions</td>
</tr>
<tr>
<td></td>
<td>Student-t</td>
</tr>
<tr>
<td>GARCH Asymmetry μ</td>
<td>0.17923</td>
</tr>
<tr>
<td>(0.06332)</td>
<td>(0.06335)</td>
</tr>
<tr>
<td>ω</td>
<td>0.35639</td>
</tr>
<tr>
<td>(0.0569)</td>
<td>(0.0569)</td>
</tr>
<tr>
<td>FIGARCH d</td>
<td>0.45012</td>
</tr>
<tr>
<td>α₁</td>
<td>0.0627</td>
</tr>
<tr>
<td>(0.05046)</td>
<td>(0.05048)</td>
</tr>
<tr>
<td>β₁</td>
<td>0.37153</td>
</tr>
<tr>
<td>(0.06904)</td>
<td>(0.06901)</td>
</tr>
<tr>
<td>ν</td>
<td>2.225481</td>
</tr>
<tr>
<td>ln(ξ)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>AIC</td>
<td>-13774.3</td>
</tr>
<tr>
<td>SIC</td>
<td>-13975.1</td>
</tr>
<tr>
<td>R.Skewness</td>
<td>0.1228</td>
</tr>
<tr>
<td>R.Kurtosis</td>
<td>6.5393</td>
</tr>
<tr>
<td>JB</td>
<td>3951.5</td>
</tr>
<tr>
<td>Q(5)</td>
<td>148.044</td>
</tr>
<tr>
<td>Q²(5)</td>
<td>1.8904</td>
</tr>
</tbody>
</table>

( ) indicates standard error, [ ] indicates p-value

Table 4.3. Estimation Results of FIGARCH Models for the RASEIN

In summary, we provide evidence of long memory in the volatility of all the
sectors of the Greek Stock market, which suggest that, all the market sector under examination are weak form inefficient. This is an evidence of violation of efficient market hypothesis, which can lead to the arbitrage opportunities for international investors who are interested to invest in Greek Stock market. Furthermore, our results confirm that the volatility has a predictable structure in all the stock market, indicating the need of improving regulatory and economic reforms within the Greek financial and economy system. From now on is easy to forecast Volatility thus, VaR values, based on FIGARCH model with Skewed Student-t (SST). From this point the can be forcasts and evaluation of them and the results of VaR can be analyzed based on Kupiec LR test. Mainly in literature provided FIGARCH and FIAPARCH models distributed the skewed student-t outperform the other symmetric distributions. It can be concluded that the volatility models with skewed student-t distribution are more suitable for VaR calculations.
Chapter 5

Conclusions

In this study we attempted to re-built all theoretical basis until the point the FIGARCH model be understandable from someone who listens to it for the very first time. Then empirically we look out for the long memory in the presence of Asymmetry in stock market volatility like the Greek financial market.

To study the stock market long memory we estimate FIGARCH model proposed by Baillie et al. (1996) using daily returns calculated by Athens Stock Market General index (ATHEX).

We find evidence of long memory. This implies that the market sector under investigation are weak form inefficient and our results show that the volatility in ASE stock market INDEX has a predictable structure.

Our results indicate the need of regulatory and economic reforms. As per our empirical investigation, FIGARCH Skewed Student-t (SST) model performs better than the FIGARCH Student-t (ST) models when applied in returns timeseries of closing prices of Athens General Index Stock Market.

Furthermore there can be VaR forecasts and evaluations, then can benchmarked and evaluated by Kupiec LR test against predictions of other models and also against actual timeseries results. Literature has shown FIGARCH and FIAPARCH models can conceive better the volatility concept in Asymmetry environments, much better, than other GARCH models as we also conclude.
Also we strongly propose that econometric software packages must update their model databases so the practitioner can fit most modern models and also make forecasts and VaR calculations more easy and fast.

Except the fact that the volatility in ASE stock market INDEX has a predictable structure proved and conceived by a FIGARCH model new evolution dictates models as the APARCH (Asymmetric Power Autoregressive Conditional Heteroscedasticity) and FIAPARCH models to have more efficient fit in Asymmetry environments. This field must continue to be a research point for econometricians so to fit most accurately models about the volatility concept under Asymmetry.
Bibliography


jhttps://doi.org/10.2469/faj.v60.n6.2669


https://ssrn.com/abstract=1100237
or
http://dx.doi.org/10.1111/j.1368-423X.2008.00229.x

es.fsv.cuni.cz/default/file/download/id/18463

https://www.cluteinstitute.com/ojs/index.php/.../8444

https://doi.org/10.1108/00215000380001141

http://www.keaipublishing.com/en/journals/jfds/

[16] Grzegorz Mentel (2013) *Parametric or Non-Parametric Estimation of Value-At-Risk*. International Journal of Business and Management; Vol. 8, No. 11; 2013 ISSN 1833-3850 E-ISSN 1833-8119 Published by Canadian Center of Science and Education
http://www.econjournals.com


http://www.keaipublishing.com/en/journals/jfds/


http://www.econjournals.com


[23] Sites

http://www.investopedia.com

http://soundmoneyproject.org/2016/05/decline-in-the-volatility-of-bitcoin/

https://www.consumerfinance.gov

http://www.naag.org

https://www.helex.gr/el/web/guest/index-historic

http://www.naftemporiki.gr/finance/athexIndices

https://finance.yahoo.com


http://www.timeseriesmodelling.com

http://gretl.sourceforge.net

[24] Class Notes

http://www.cims.nyu.edu/ almgren/timeseries/VolForecast2.pdf

-N. Sariannidis, Econometrics, ATEI Western Macedonia